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Math 231:Introduction to Ordinary Differential Equations
Mini-Project: Relative Infectivity, Average time of Infectivity, and the Spread of Disease

Consider the spread of a viral disease through an isolated population. Suppose that the following assumptions are valid:

a. Individuals are infected at a rate proportional to the product of the number of infected and susceptible individuals. The constant of proportionality is denoted λ which is a measure of the relative infectivity of the disease, and is roughly given by the fraction of encounters per day between a susceptible person and an infected person that result in the infection of the susceptible person.

b. The length of the incubation period is negligible. That is, infected individuals are immediately infectious.

c. On the average, an infected individual dies or recovers after n days. This is also referred to as the average time of infectivity. Of those individual that die or recover per week, the fraction $0 \leq r \leq 1$ of them recover and again become susceptible to the disease. The remaining $(1 - r)$ fraction of the individuals that were infected but are no longer, have either died or gained immunity to the disease.

d. Infected individuals do not give birth, but susceptible individuals have a net birth/death rate (due to other causes) in the United States of 0.005 per individual per year. Newborns are susceptible.

We can then model the course of the disease by letting $S(t)$ represent the number of susceptible individuals and $I(t)$ the number of infected at time t. Then:

$$\frac{dS}{dt} = -\lambda SI + .005S + \frac{r}{n}I$$

$$\frac{dI}{dt} = \lambda SI - \frac{1}{n} I$$

My intention for a group of three people is that numbers 1 and 5 are done as a group, while each member takes primary responsibility for one of 2,3, or 4.

1. Explain the model - describe what each term of the right hand sides of the equations represents and why it is included. Why might it be okay that we've used an exponential growth term for the susceptible population, rather than a logistic growth term?

2. First let the probability of infection resulting from contact with an infected be 0.1% so that roughly $\lambda = 0.001$. Take $n = 10$ days, and $r = 0.9$. Do a full phase plane analysis, give equillibria, and interpret your results in terms of the possiblities for the dynamics of this disease over time.

   Choose 3 different sets of initial conditions, each from a different region of the phase plane and use MATLAB to solve for $S(t)$ and $I(t)$ over time in each case. Discuss the relevance of your results to the physical problem at hand. What are the potential outcomes? Can you think of a disease that this may be relevant for modeling?

   Now set $r = 0.2$, leaving n and λ the same. What would this represent? Repeat the steps above for this new situation. What are the potential outcomes? Can you think of a disease this might be relevant for modeling? Compare the results of this case with the results you obtained for $\lambda = .001$. Explain the differences in dynamics in terms of the physical changes in the disease.

   Finally use MATLAB to explore what happens when we instead let $r = 0$ (you don't need to construct the phase plane for this, just use the same inital conditions you did for the case of $r = 0.2$). Again, explain the differences in dynamics in terms of the physical changes in the disease.

3. For this part, we will investigate how the birth rate affects the spread of our disease. In the United States we have a net birth/death rate of 5 people per thousand in a year. But in Algeria, the next birth death rate is 19 people per thousand in a year (so .019). Replace .005 with .019 in the model above, and do a full phase plane analysis with the infectivity rate set again at 0.1%, or $\lambda = .001$ and the average length of infection again being $n = 10$, and $r = 0.9$. Give equillibria and interpret your results in terms of the spread of this disease. How does changing the birth rate affect the disease dynamics, if at all, and why do these changes occur?

Choose 3 different sets of initial conditions, each from a different region of the phase plane and use MATLAB to solve for $S(t)$ and $I(t)$ over time in each case. Discuss the relevance of your results to the physical problem at hand. What are the potential outcomes?

Now keep the growth rate at .019, but set $r = 0.2$ and repeat the steps above. What changes do you see in the potential outcomes if any? How does this compare with the results in part (a) for the same r? What determines what sort of outcome we'll actually see in the long run?

4. For this part, we will investigate how the likelihood of disease transmission affects the dynamics of our disease. Rather than $\lambda = 0.001$, do a full phase plane analysis with $\lambda = 0.1$, keep $r = 0.9$, and let the net growth rate again be .005 and $n = 10$. Interpret your results in terms of the spread of this disease.

   Choose 3 different sets of initial conditions, each from a different region of the phase plane, if possible, and use MATLAB to solve for $S(t)$ and $I(t)$ over time in each case. Discuss the relevance of your results to the physical problem at hand. What are the potential outcomes? Again, compare the results for this case to those obtain in problem one where $\lambda = 0.001$, and all other parameters are the same. Explain the differences you see in the dynamics in terms of the changes in the physical system reflected by the change in λ.

   Finally repeat the steps above for $\lambda = 0.0001$, with all other parameters held the same.

5. Summarize your findings from problems 2,3,and 4 in terms of what they indicate about how r, λ, and population growth rate effect the spread of disease. In light of your findings, is there a particular parameter that seems to have a stronger effect on the long-term persistance level of the disease? If so, what sort of measures would potentially then be most effective in combating a given disease?