1. [3 point] Give the order, state which is the independent/dependent variable, and why the differential equation is or is not linear:

$$\frac{dx}{dt} = k(4-x)(1-x)$$

first order
independent variable is $t$
dependent variable is $x$

this eqn is nonlinear because $x$ (the dep. var.) is combined with itself multiplicatively — i.e. the term $x^2$ is on the right hand side.

2. [1 point] Write a differential equation that fits the physical description: The rate of change of the mass $A$ of salt at time $t$ is proportional to the square of the mass of salt present at time $t$.

$\frac{dA}{dt} = kA^2$ where $k$ is a proportionality constant

3. [3 points] Determine whether $y = \sin(x) + x^2$ is a solution to the differential equation $\frac{d^2y}{dx^2} + y = x^2 + 2$. Show why or why not.

$y = \sin(x) + x^2$
$\Rightarrow y' = \cos(x) + 2x$
$\Rightarrow y'' = -\sin(x) + 2.$

$\Rightarrow \frac{d^2y}{dx^2} + y = x^2 + 2$

becomes:

$(-\sin(x) + 2) + (\sin(x) + x^2) = x^2 + 2$

$\Rightarrow x^2 + 2 = x^2 + 2.$

This equality holds for all $x$, so $y = \sin(x) + x^2$ is a solution.

4. [3 points] Determine whether $x^2 + y^2 = 4$ is an implicit solution to $\frac{dy}{dx} = \frac{x}{y}$. Show why or why not.

implicit diff $\Rightarrow \frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(4) \Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}.$

since there is no interval on which $-\frac{x}{y} = \frac{x}{y}$

$x^2 + y^2 = 4$ is not a soln.