

Homotopy transversality thm.

$f: X \rightarrow Y$ $\partial Y = \emptyset$, $Z \subset Y$ w/o bdy.

$\Rightarrow \exists g: X \rightarrow Y$ s.t. $g \sim f$ (homotopic)

$g \pitchfork Z$, $\partial g \pitchfork Z$

(g can be taken in an arb. $W^1(X, Y)$ nbd. of f)

Transversality extension thm.

$f: X \rightarrow Y$ diff'ble, $Z \subset Y$ submf w/o bdy $\partial Y = \emptyset$

$C \subset X$ closed s.t. $f|_C \pitchfork Z$, $\partial f|_C \pitchfork Z$.

$\Rightarrow \exists g: X \rightarrow Y$ $g \sim f$, $g \pitchfork Z$, $\partial g \pitchfork Z$ s.t. $g|_C = f$.

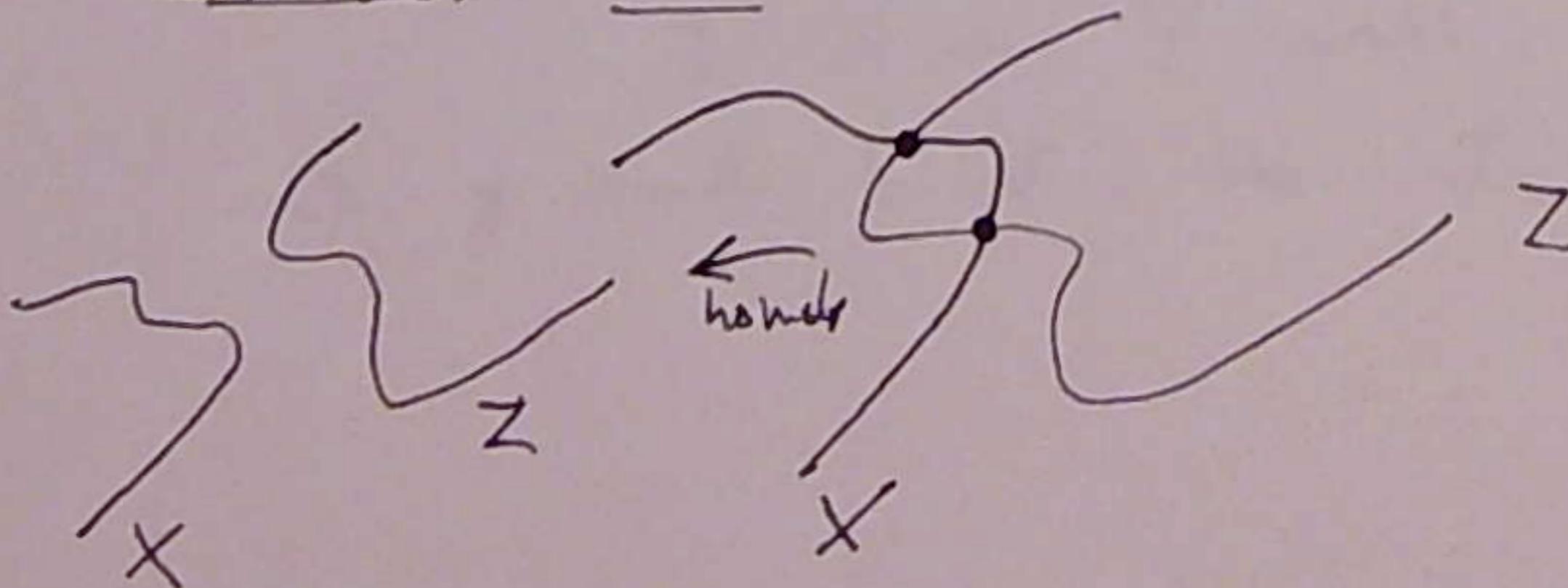
Pf: see [G-P]

Ex. $C = \partial X$

If $h: \partial X \rightarrow Y$ (diff'ble) transv to Z

and extends to $f: X \rightarrow Y$, then it extends to $f \pitchfork Z$ ($\text{in } X$).

In part If $f: X \rightarrow Y$ s.t. $\partial f \pitchfork Z$, then $\exists g: X \rightarrow Y$ homotopic to f , $g \pitchfork Z$ and $\partial f = \partial g$

Mod Z homotopy invariantsIntersection number

$X, Z \subset Y$ submf

$$\dim X + \dim Z = \dim Y$$

If $X \pitchfork Z$ on Y

$$\dim (X \cap Z) = 0$$

(isolated pts)

Def $f: X \rightarrow Y$ diff., transv. to Z ($Z \subset Y$, both w/o body).
 Assume $\dim X + \dim Z = \dim Y$.

$f^{-1}(Z)$: 0-dim'l submanifold of X (isolated pt.).

$I_2(f, Z) \stackrel{def}{=} \# f^{-1}(Z) \pmod{2}$ intersection number.

Thm

$f_0, f_1: X \rightarrow Y$ homotopic, transv. to Z

$$\Rightarrow I_2(f_0, Z) = I_2(f_1, Z).$$

Pf.

$F: X \times I \rightarrow Y$ homoty from $f_0 \rightarrow f_1$.

we may assume $F \pitchfork Z$ on $X \times I$. (By the exten. th.)

(also $\partial F \pitchfork Z$, since it is f_0, f_1)

$F^{-1}(Z)$ is a submanifold of $X \times I$ of dimension 1

w/ body $\partial [F^{-1}(Z)] = f_0^{-1}(Z) \times \{0\} \sqcup f_1^{-1}(Z) \times \{1\}$.

$\#(\partial \text{ of 1 dim'l manifold})$ is even (from the defn.).

$$\therefore \# f_0^{-1}(Z) + \# f_1^{-1}(Z) = 0 \pmod{2}$$

$$\therefore I_2(f_0, Z) = I_2(f_1, Z) \pmod{2}$$

Ex

bdry thm. $X = \partial W$ W pt.

$g: X \rightarrow Y$ smth. $Z \subset Y$ submfld

If g extends to W , then $I_2(g, Z) = 0$