MATH 561 - HOMEWORK 1

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4. If X is first countable and $a \in E'$ then one may find a sequence $(x_n)_{n \geq 1}$ in E, so that $\lim x_n = a$.

Solution. Let $\{U_n \mid n \in \mathbb{N}\}$ be a local basis at $a \in E'$. Define $V_n = \bigcap_{i=1}^{n} U_i$. Every V_i is open since it is a finite intersection of open sets. Additionally, the set $\{V_i \mid i \in \mathbb{N}\}$ is a local basis at a since for every open neighborhood U of a, there exists some $U_n \subset U$, but then $V_n \subset U_n$ and so $V_n \subset U$. Lastly, we note that $V_i \supset V_{i+1}$ for all $i \in \mathbb{N}$. Now every neighborhood V_n contains some point $a \neq x_n \in E$. Then (x_n) is a sequence in E and it converges to a since for every open neighborhood V of a, there exists some $N \in \mathbb{N}$ so that $V_n \subset V$ for all $n \geq N$, implying $x_n \in V$ for all $n \geq N$. $\square$