M 341 - Spring 2013
Exam1 - solutions.

1. If $\lim a_n = L$, $\exists N \in \mathbb{N}$ s.t. $|a_n - L| < 1$ for all $n \geq N$ (so $|a_n| \leq |L|+1$)

Then letting $M = \max\{|a_1|, |a_2|, \ldots, |a_N|, |L|+1\}$ we have $|a_n| \leq M$ $\forall n \geq 1$.

2. $|a_n b_n| \leq M|b_n|$ if $|a_n| \leq M$. Since $M\sum_{n=1}^{\infty} |b_n|$ converges, then also $\sum_{n=1}^{\infty} |a_n b_n|$ converges (by comparison).

3. (i) False : $\sup\{1,2\} = 2$, but 2 is not a limit point of $A = \{1,2\}$.

(ii) True : if $x \in \overline{A}$, then $x = \lim_n x_n$ for a sequence $(x_n)_{n\geq 1}$, $x_n \in A$ $\forall n$.

Thus $x_n \leq M$ $\forall n \geq 1 \implies x \leq M$.

4. (i) $\text{int}(A) = \{ x \in A \mid (\exists \delta > 0)(V_\delta(x) \subseteq A) \}$

(ii) $A = [0,1)$, $B = [1,2]$, $A \cup B = [0,2]$ $\text{int}(A \cup B) = (0,2)$

$\text{int}(A) = (0,1)$ $\text{int}(B) = (1,2)$ $\text{int}(A) \cup \text{int}(B) = (0,1) \cup (1,2)$

5. Let $x_n \in F \setminus A$, assume $x_n \longrightarrow x \in \mathbb{R}$.

Since $x_n \in F$ and F is closed, also $x \in F$.

We claim $x \notin A$. Otherwise, since A is open, $V_\varepsilon(x) \subseteq A$ for some $\varepsilon > 0$. But then $x_n \in V_\varepsilon(x)$ for some N and all $n \geq N$, (since $\lim x_n = x$) contradicting $x_n \notin A$. This proves the claim.

Thus $x \in F \setminus A$, so $F \setminus A$ is closed.