MATH 341, ANALYSIS I-FIRST TEST, Feb. 18, 2013.

Instructions. Closed book, closed notes, no electronics. Time given: 50 min. 20 pts per problem. *Full details for full credit.*

1. Using only the definition of $\lim a_n = L$, show that convergent sequences are bounded.

2. Let $(a_n), (b_n)$ be sequences. Suppose (a_n) is bounded and the series Σb_n converges absolutely. Prove that the series $\Sigma a_n b_n$ converges absolutely.

3. *True or False?* If true give a short proof, if false give a counterexample.

(a) If $A \subset \mathbb{R}$ is nonempty and bounded above, $\sup(A)$ is a limit point of A.

(b) If $A \subset \mathbb{R}$ is nonempty and bounded above, then so is its closure $\overline{A}$.

4. (i) Let $A \subset \mathbb{R}$ be non-empty. Give the definition of $int(A)$ (the interior of A) *without* using the words "interior point" (unless you define that, too.)

(ii) If $A, B \subset \mathbb{R}$ are both non-empty, $int(A) \cup int(B) \subset int(A \cup B)$ (You're *not* being asked to prove this). Give an example to show equality does not always hold.
Hint: it is enough to consider intervals.

5. Let F, A be nonempty subsets of $\mathbb{R}$. If F is closed and A is open, prove that $F \setminus A$ is closed.