Math 251, spring 2010 – Exam 3 slns.

1. $\lambda^2+2\lambda+5=0 \longrightarrow \lambda=-1\pm 2i$ $\qquad E(-1+2i)\begin{cases}-x_1-4x_2=-x_1+2ix_1\\ \cdots\end{cases}$

$\Rightarrow B=\begin{bmatrix}2&0\\0&-1\end{bmatrix}$ $\quad \Lambda=\begin{bmatrix}-1&2\\-2&-1\end{bmatrix}=\underset{r>0}{\sqrt5}\underbrace{\begin{bmatrix}-1/\sqrt5&2/\sqrt5\\-2/\sqrt5&-1/\sqrt5\end{bmatrix}}_{R\in\mathbb{O}_2}$ $\Rightarrow 2x_2=-ix_1,\ v=\begin{bmatrix}2\\-i\end{bmatrix}=\begin{bmatrix}2\\0\end{bmatrix}+i\begin{bmatrix}0\\-1\end{bmatrix}$

2. $A-2\mathbb{I}=\begin{bmatrix}-2&1&0\\-4&2&0\\3&-1&-1\end{bmatrix}$ $\begin{cases}-2x_1+x_2=0 \longrightarrow x_2=2x_1\\ 3x_1-x_2-x_3=0 \longrightarrow x_1=x_3\end{cases}$ $(1,2,1)$ $\qquad \Lambda=\begin{bmatrix}1&&\\&2&1\\&0&2\end{bmatrix}$

$(A-2\mathbb{I})\,w=(1,2,1)$ $\begin{cases}-2x_1+x_2=0 \longrightarrow x_2=2x_1\\ 3x_1-x_2-x_3=1 \longrightarrow x_1-x_3=1\end{cases}$ $w=(1,2,0)$ $\qquad B=\begin{bmatrix}0&1&1\\0&2&2\\1&1&0\end{bmatrix}$

$A\,e_3=e_3 \longrightarrow E(1)=\langle e_3\rangle$

3. $A=\begin{bmatrix}-2&0&-36\\0&-28&0\\-36&0&-48\end{bmatrix}$ $\qquad A+3\mathbb{I}=\begin{bmatrix}1&0&-36\\0&0&0\\-36&0&-20\end{bmatrix}\longrightarrow E(-3)=\langle 0,1,0\rangle$

$A-25\mathbb{I}=\begin{bmatrix}-27&0&-36\\0&-28&0\\-36&0&-48\end{bmatrix}\longrightarrow \left.\begin{matrix}3x_1+4x_3=0\\ x_2=0\end{matrix}\right\}$ $E(25)=\langle 4,0,-3\rangle \overset{P.A.T.}{\Longrightarrow} E(5)=\langle 3,0,4\rangle$

$\Lambda=\operatorname{diag}(25,-50,-3)$ $\qquad P=\frac{1}{5}\begin{bmatrix}4&3&0\\0&0&5\\-3&4&0\end{bmatrix}$

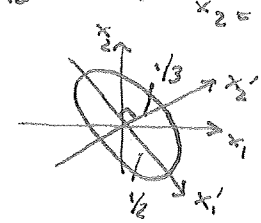
4. $\operatorname{tr}A=13,\ \det A=36$

$A=\begin{bmatrix}5&-2\\-2&8\end{bmatrix}\longrightarrow \lambda=4,9$ $\qquad \min_S Q=4,\ \max_S Q=9$

$E(4)=\langle(2,1)\rangle,\ E(9)=\langle(-1,2)\rangle$ $\qquad x=Px',\ P=\frac{1}{\sqrt5}\begin{bmatrix}2&-1\\1&2\end{bmatrix}$ $\qquad x_1=\frac{2}{\sqrt5}x_1'+\frac{1}{\sqrt5}x_2'$

$Q'(x',x')=4(x_1')^2+9(x_2')^2=1$ $\qquad x_2=-\frac{1}{\sqrt5}x_1'+\frac{2}{\sqrt5}x_2'$

$\longrightarrow$ ellipse w/ principal axes $\underset{\downarrow}{x_1'}$ $a=1/2$ $\quad \underset{\downarrow}{x_2'}$ $b=1/3$



5. $\Sigma=\begin{bmatrix}1&&\\&1&\\&&2\end{bmatrix}$ $\qquad Av_1=v_1=u_1$

$Av_2=v_2=u_2$

$Av_3=2u_3=(-2)(-u_3)\longrightarrow u_3=-v_3$

$\begin{cases}v_1=(1,0,-1)/\sqrt2\\ v_2=(1,-4,1)/3\sqrt2\\ v_3=\frac13(2,1,2)\end{cases}$

$U=\begin{bmatrix}1/\sqrt2&1/3\sqrt2&-2/3\\0&-4/3\sqrt2&-1/3\\-1/\sqrt2&1/3\sqrt2&-2/3\end{bmatrix}$ $\qquad V=\begin{bmatrix}1/\sqrt2&1/3\sqrt2&2/3\\0&-4/3\sqrt2&1/3\\-1/\sqrt2&1/3\sqrt2&2/3\end{bmatrix}$

6. $\begin{matrix}Av=\lambda v\\ Aw=\mu w\end{matrix}\longrightarrow \lambda(v\cdot w)=Av\cdot w\underset{\text{since } A=A^t}{=}v\cdot Aw=\mu(v\cdot w)\Rightarrow(\lambda-\mu)(v\cdot w)=$

$\Downarrow \lambda\neq\mu$

$v\cdot w=0$