Math 251, spring 2010- Homework set 7 (due TUESDAY, Mar. 23.) (One point per item.)

1. (i) Find the images of the standard basis vectors of $\mathbb{R}^3$ under reflection on the hyperplane $x_1 + 2x_2 + 3x_3 = 0$. (ii) Write down the matrix of this reflection in the standard basis.

2. (i) Use the Gram-Schmidt procedure to find an orthonormal basis of the subspace $\{x_1 + 2x_2 + x_4 = 0\}$ of $\mathbb{R}^4$. (ii) Write down the matrix (in the standard basis) of orthogonal projection onto this subspace.

3. Find the 3×3 orthogonal matrix that implements the rotation R in $\mathbb{R}^3$ with axis $\langle(2, 2, 1)\rangle$, by an angle of $\pi/3$ radians (counterclockwise when looking down the axis). (*Remark.* Give your answer in the form $R = P[R]_{\mathcal{B}}P^t$, where the matrices $[R]_{\mathcal{B}}$ (of R in an 'adapted basis') and P (orthogonal) are explicitly given; you don't need to perform the matrix multiplication.)

4. Find the least squares solution of $Ax = b$, and the orthogonal projection of b onto the column space of A:

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ -1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}.$$

5. Find the orthogonal projection of $(1, 2, 3)$ onto the subspace of $\mathbb{R}^3$ spanned by $v_1 = (1, 0, 1), v_2 = (1, 2, 1)$.

6. Find the orthogonal projection of $(1, 2, 3, 4)$ onto the two-dimensional subspace of $\mathbb{R}^4$ with defining equations:

$$x_1 + x_2 + x_3 = 0, \quad x_2 + 2x_3 + x_4 = 0.$$

7. Find the least-squares straight line fit to the four points: $(0, 1), (2, 0), (3, 1), (4, 5)$.

8. Find the quadratic polynomial that best fits the five points $(-1, 3), (0, -1), (1, 2), (2, 3), (3, 7)$.