Math 251, spring 2010- Homework set 6 (due Thursday, Mar. 18.) (One point per item, 9 items.)

1. Find an A=LU decomposition for A (L lower-, U upper-triangular, both invertible.)

\[
A = \begin{bmatrix}
3 & -6 & -3 \\
2 & 0 & 6 \\
-4 & 7 & 4
\end{bmatrix}.
\]

2. P is the orthogonal projection onto the subspace V of \(\mathbb{R}^4\): \(V = \langle (1,1,2,0), (2,0,1,1) \rangle\). Find (i) an orthonormal basis for V; (ii) the matrix of P in the standard basis.

3. (i) Use the Gram-Schmidt procedure to find an orthonormal basis of the subspace \(\{x_1 + 2x_2 + x_4 = 0\}\) of \(\mathbb{R}^4\). (ii) Write down the matrix (in the standard basis) of orthogonal projection onto this subspace.

4. Find the \(A = OT\) decomposition of the matrix:

\[
A = \begin{bmatrix}
1 & 1 \\
-2 & 1 \\
2 & 1
\end{bmatrix},
\]

where \(O \in M_{3 \times 2}\) has orthonormal columns and \(T \in \mathbb{T}_2\). (That is, find \(O\) and \(T\).)

5. Show that if \(\mathcal{E} = \{e_1, \ldots, e_r\}\) is an orthonormal basis for a subspace \(V\) of \(\mathbb{R}^n\), then for any \(v \in V\):

\[
|v|^2 = (v \cdot e_1)^2 + (v \cdot e_2)^2 + \ldots (v \cdot e_r)^2.
\]

(Recall \(|v|^2 = v \cdot v\).)

6. Let \(\mathcal{B} = \{e_1, \ldots, e_n\}\) be an orthonormal basis of \(\mathbb{R}^n\), ‘adapted’ to a subspace \(V\) of dimension \(r < n\) (in the sense that \(V = \langle e_1, \ldots, e_r \rangle\)). Let \(P_V : \mathbb{R}^n \to \mathbb{R}^n\) be the orthogonal projection onto \(V\).

(i) Write down the matrix of \(P_V\) in the basis \(\mathcal{B}\);

(ii) Show that the matrix of \(P_V\) in the standard basis is a symmetric matrix. (*Hint: note that the transition matrix \(B\) satisfies \(B^{-1} = B^t\)-why?)