Math 251, spring 2010- Homework set 6 (due Thursday, Mar. 18.) (One point per item, 9 items.)

1. Find an A=LU decomposition for A (L lower-, U upper-triangular, both invertible.)

$$A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix}.$$

2. P is the orthogonal projection onto the subspace V of $\mathbb{R}^4$: $V = \langle (1,1,2,0),(2,0,1,1)\rangle$. Find (i) an orthonormal basis for V; (ii) the matrix of P in the standard basis.

3. (i) Use the Gram-Schmidt procedure to find an orthonormal basis of the subspace $\{x_1 + 2x_2 + x_4 = 0\}$ of $\mathbb{R}^4$. (ii) Write down the matrix (in the standard basis) of orthogonal projection onto this subspace.

4. Find the $A = OT$ decomposition of the matrix:

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 2 & 1 \end{bmatrix},$$

where $O \in \mathbb{M}_{3\times 2}$ has orthonormal columns and $T \in \mathbb{T}_2$. (That is, find O and T.)

5. Show that if $\mathcal{E} = \{e_1, \ldots, e_r\}$ is an orthonormal basis for a subspace V of $\mathbb{R}^n$, then for any $v \in V$:

$$|v|^2 = (v \cdot e_1)^2 + (v \cdot e_2)^2 + \ldots (v \cdot e_r)^2.$$

(Recall $|v|^2 = v \cdot v$.)

6. Let $\mathcal{B} = \{e_1, \ldots, e_n\}$ be an orthonormal basis of $\mathbb{R}^n$, 'adapted' to a subspace V of dimension $r < n$ (in the sense that $V = \langle e_1, \ldots, e_r\rangle$). Let $P_V : \mathbb{R}^n \to \mathbb{R}^n$ be the orthogonal projection onto V.

(i) Write down the matrix of P_V in the basis $\mathcal{B}$;

(ii) Show that the matrix of P_V in the standard basis is a symmetric matrix. (*Hint:* note that the transition matrix B satisfies $B^{-1} = B^t$-why?)