1. $A$ is a $4 \times 3$ matrix, and the reduced row-echelon forms of $A$ and $A^t$ are given:

$$A_{rref} = \begin{bmatrix} 1 & 0 & 7/5 \\ 0 & 1 & 1/5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^t_{rref} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

(i) Find a set of defining equations for $\text{Ran}(A)$ (recall $\text{Ran}(A)^\perp = \text{Ker}(A^t)$).

(ii) Find a matrix $B$ with the same kernel and the same range as $A$.

2. Use row reduction to compute $A^{-1}$ (or to show $A$ is not invertible).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 0 & 1 \end{bmatrix}.$$

**Remark:** you must show the row-reduction steps for credit in this problem.

3. Let $A$ be a $4 \times 3$ matrix. Suppose $A' \in M_{4\times3}$ is obtained from $A$ as a result of the following operations on the columns $C_1, C_2, C_3$ of $A$:

$$C_1(A') = C_1 - 2C_2 + C_3, \quad C_2(A') = 2C_2 - 5C_3, \quad C_3(A') = C_3 + 3C_2.$$ 

Find a $3 \times 3$ matrix $C$ (invertible) so that $A' = AC$.

**Hint:** use the “column description” of matrix multiplication. In class we saw an analogous example involving operations on the rows of $A$, and multiplication by a matrix on the left, $A' = RA$.

4. Let $\{e_1, \ldots, e_4\}$ be the standard basis of $\mathbb{R}^4$, $\{f_1, f_2, f_3\}$ the standard basis of $\mathbb{R}^3$. Using the information below, find the matrix $AB$, and its rank.

$$B(e_1) = (-1, 0, 3), \quad B(e_2) = (2, 3, -2), \quad B(e_3) = (1, 1, -1), \quad B(e_4) = (0, 1, -1),$$

$$A(f_1) = (1, -1), \quad A(f_2) = (2, -2), \quad A(f_3) = (-1, 1).$$

5. Let $A \in M_{m\times n}, C \in M_{n\times n}$.

(i) Explain why we always have $\text{Col}(AC) \subseteq \text{Col}(A)$.

(ii) Explain why, if $C$ is invertible, we also have:

$$v \in \text{Ran}(A) \Rightarrow v \in \text{Ran}(AC),$$

so that in fact: $\text{Ran}(AC) = \text{Ran}(A)$ in this case.

**Hint.** In class we saw the corresponding statement for the kernel. In the present case, use: $Ax = ACC^{-1}x$ for all $x \in \mathbb{R}^n$ and the definition of range.