Math 251, spring 2010- Homework set 3 (due Thursday, Feb.11.)

1. Given the matrix $A$ and a row-echelon form $A_{\text{ref}}$ of $A$, find bases for the four subspaces $\text{Ran}(A)$, $\text{Ran}(A^t)$, $\text{Ker}(A)$, $\text{Ker}(A^t)$.

$$
A = \begin{bmatrix}
1 & -1 & 2 & 4 \\
2 & 1 & 0 & -1 \\
1 & 1 & 0 & -1 \\
4 & 3 & 0 & -3 \\
0 & -2 & 2 & 5
\end{bmatrix}, \quad A_{\text{ref}} = \begin{bmatrix}
1 & -1 & 2 & 4 \\
0 & 3 & -4 & -9 \\
0 & 0 & 2 & 3 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

2. For the matrix $A$ in problem 1: find all (independent) linear relations among its rows, and among its columns.

3. Find the general solution to the linear system $Ax = b$, for $b = (1, -1, 2, 3, 1)$. $A$ is given in problem 1.

4. Find the reduced row-echelon form $A_{\text{rref}}$ of the matrix $A$ in problem 1.

5. (i) Find a basis for the subspace of $\mathbb{R}^5$:

$$
E = \langle (1, -1, 2, 3, 1), (1, 2, 1, -2, -1), (3, 3, 4, -1, -1), (1, -4, 3, 8, 3) \rangle
$$

(ii) Decide whether the vector $v = (1, 8, -1, -12, -5)$ is in $E$.

6. Find a basis for the orthogonal complement of the subspace $E$ of $\mathbb{R}^5$ in problem 5.

7. Find a basis for $U^\perp \subset \mathbb{R}^5$, where $U$ is the subspace of $\mathbb{R}^5$ given by the defining equations below (note: maybe the equations are not independent!)

$$
\begin{align*}
&x_1 + 3x_2 - x_3 + x_4 + x_5 = 0 \\
x_1 - x_2 + 2x_3 + 3x_4 + x_5 = 0 \\
x_1 + 11x_2 - 7x_3 - 3x_4 + x_5 = 0
\end{align*}
$$

8. Show that if $A \in \mathbb{M}_n$ is symmetric ($A^t = A$) and $b \in \mathbb{R}^n$ is any vector then: the system $Ax = b$ is consistent if (and only if) $b \cdot v = 0$ for any $v \in \mathbb{R}^n$ such that $Av = 0$. (Use the definitions of Kernel, Range and orthogonal subspaces).