**Math 251, fall 2009-Optional test. Date: 11/4/2009.** Justify your answers for credit. Calculators allowed. Closed book, closed notes. Time given: 50 min.

**1.** Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator with eigenvalues $x = 1/2$ and $x = 5$, and corresponding eigenspaces:

$$E(1/2) = \langle (2,1,1) \rangle, \quad E(5) = \langle (2,1,1) \rangle^{\perp}.$$

Find matrices $P$ and $A$ in $GL_3$ so that the matrix of $T$ in the standard basis of $\mathbb{R}^3$ has the form: $[T]_{std} = PAP^{-1}$.

**2.** $P$ is the projection onto the subspace $V = \{x_1 + x_2 + 2x_3 = 0\}$ of $\mathbb{R}^3$, parallel to the subspace $W = \langle (1,2,2) \rangle$.(i)[14] Find $3 \times 3$ matrices $B$ (invertible) and $\Lambda$ so that the matrix of $P$ in the standard basis is $B\Lambda B^{-1}$ (ii)[6] What are the eigenvalues/eigenspaces of $P$? Justify.

**3.** (i) Use the Gram-Schmidt procedure to find an orthonormal basis of the subspace $\{2x_1 + x_2 - x_3 = 0\}$ of $\mathbb{R}^3$. (ii) Find a matrix $A$ so that the matrix $P$ (in the standard basis) of orthogonal projection onto this subspace is given by $P = AA^t$.

**4.** Given the following three-dimensional subspaces of $\mathbb{R}^4$, compute the dimension of their intersection and find a basis for it.

$$V : x_1 + 2x_2 + 3x_3 = 0 \qquad W : x_1 - 2x_3 - x_4 = 0.$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 0 & 0 & 3 & 4 \\ 4 & 4 & 0 & 5 & 2 \end{bmatrix}$$

**5.** Find the general solution of the system $Ax = b$, $b = (1,1,3)$.(*Hint:* begin by reducing the augmented matrix to row-echelon form, then find a basis for the nullspace of $A$.)