Math 251, fall 2009-Exam 1. Date: 9/25/2009. Justify your answers for credit. Calculators allowed. Closed book, closed notes. Time given: 50 min.

1. For the 3×5 matrix A given below, find:

(i) Conditions on the coordinates (b_1, b_2, b_3) of b so that the system $Ax = b$ has at least one solution $x \in \mathbb{R}^5$;

(ii) The general solution of $Ax = b$ if $b = (1, 1, 1)$.

$$A = \begin{bmatrix} 2 & -1 & 0 & 3 & 2 \\ -1 & 5 & 6 & 9 & 2 \\ 1 & 1 & 2 & 5 & 2 \end{bmatrix}.$$

2. Let V be the subspace of $\mathbb{R}^5$ defined by the l.i. system of equations:

$$V \begin{cases} 2x_1 + x_3 - x_5 & = & 0 \\ x_2 - x_4 - x_5 & = & 0 \end{cases}$$

Find a basis for V and a basis for the orthogonal complement $V^{\perp}$.

3. Determine if the vector v is in the subspace $U \subset \mathbb{R}^4$ or not; if it is, express it as a linear combination of the given spanning set of U.

$$U = \langle (-2, 1, -1, 0), (0, 1, 0, 1), (1, 2, -1, 1) \rangle; \quad v = (-4, 2, 1, 3).$$

4. Find a 3×3 matrix A with the given row space and range:

$$Row(A) = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 = 0\}; \quad Ran(A) = \{(y_1, y_2, y_3) | y_3 = 2y_1 + y_2\}.$$

5. (i) Consider an *arbitrary* 5×3 matrix A. Is the system $Ax = b$ consistent for all vectors $b \in \mathbb{R}^5$? When consistent, are solutions guaranteed to be unique? (Justify your answers based on the possible dimensions of $Ker(A)$ and $Ran(A)$.)

(ii) Now consider a *'randomly chosen'* matrix A, also 3×5 (this implies its rank is as large as possible). If a vector $b \in \mathbb{R}^5$ is chosen 'at random', is the system $Ax = b$ likely to be consistent? When consistent, are solutions guaranteed to be unique? (Again, justify based on $Ker(A)$ and $Ran(A)$).