1. For the $3 \times 5$ matrix $A$ given below, find:
   (i) Conditions on the coordinates $(b_1, b_2, b_3)$ of $b$ so that the system $Ax = b$ has at least one solution $x \in \mathbb{R}^5$;
   (ii) The general solution of $Ax = b$ if $b = (1, 1, 1)$.

   $$A = \begin{bmatrix} 2 & -1 & 0 & 3 & 2 \\ -1 & 5 & 6 & 9 & 2 \\ 1 & 1 & 2 & 5 & 2 \end{bmatrix}.$$ 

2. Let $V$ be the subspace of $\mathbb{R}^5$ defined by the l.i. system of equations:

   $$V \left\{ \begin{array}{c} 2x_1 + x_3 - x_5 = 0 \\ x_2 - x_4 - x_5 = 0 \end{array} \right.$$ 

   Find a basis for $V$ and a basis for the orthogonal complement $V^\perp$.

3. Determine if the vector $v$ is in the subspace $U \subset \mathbb{R}^4$ or not; if it is, express it as a linear combination of the given spanning set of $U$.

   $$U = \langle (-2, 1, -1, 0), (0, 1, 0, 1), (1, 2, -1, 1) \rangle; \quad v = (-4, 2, 1, 3).$$

4. Find a $3 \times 3$ matrix $A$ with the given row space and range:

   $$\text{Row}(A) = \{(x_1, x_2, x_3) | x_1 + x_2 + x_3 = 0\}; \quad \text{Ran}(A) = \{(y_1, y_2, y_3) | y_3 = 2y_1 + y_2\}.$$ 

5. (i) Consider an arbitrary $5 \times 3$ matrix $A$. Is the system $Ax = b$ consistent for all vectors $b \in \mathbb{R}^5$? When consistent, are solutions guaranteed to be unique? (Justify your answers based on the possible dimensions of Ker($A$) and Ran($A$).

   (ii) Now consider a ‘randomly chosen’ matrix $A$, also $3 \times 5$ (this implies its rank is as large as possible). If a vector $b \in \mathbb{R}^5$ is chosen ‘at random’, is the system $Ax = b$ likely to be consistent? When consistent, are solutions guaranteed to be unique? (Again, justify based on Ker($A$) and Ran($A$)).