Linear Algebra in the grand scheme of things

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Mathematical reasoning falls into four distinct 'modes', corresponding to the four different 'trunks' of the subject:

The Four Modes of Mathematics

Algebra (algorithms)
Analysis (approximation)
Geometry (pictures)
Probability (randomness)

Some branches of mathematics

Basic Mathematics

Algebraic geometry
Number theory
Topology
Differential Geometry
Dynamical systems
Stochastic processes

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Applied Mathematics

Partial Differential Equations
Fluid mechanics
Graph theory
Coding and information theory
Mathematical statistics
General Relativity

The role of linearity

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There are many areas of mathematics (including many of engineering interest) in which the *only* classes of models mathematically understood at present are linear ones; more general ones are reduced to the linear case by approximation (*linearization*), which often works only in 'small input' or 'perturbation from exact' situations.