MATH 251, FALL 2009 (Freire)- HOMEWORK SET 2

The matrix $A$ below is to be used for problems 1,2 and 3.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & -1 \\ 2 & 0 & 0 & 3 & 4 \\ 4 & 4 & 0 & 5 & 2 \\ -1 & 2 & 0 & -2 & -5 \end{bmatrix}$$

1. Find rank(A), bases for $\text{Ker}(A)$, $\text{Row}(A)$, $\text{Ran}(A)$

2. Compute the conditions on $b$ for solvability of $Ax = b$ and the form of the general solution.

3. Find the general solution of the system $Ax = b$, $b = (1,1,3,0)$

4. Find a basis for the subspace of $\mathbb{R}^5$ defined by the equations below, and for its orthogonal complement.

$$2x_1 + x_4 + x_5 = 0, \quad x_1 - x_2 - x_3 = 0, \quad 2x_2 + 2x_3 + x_4 + x_5 = 0.$$ 

5. Given the following three-dimensional subspaces of $\mathbb{R}^4$, compute the dimension of their intersection and find a basis for it.

$$V : x_1 + 2x_2 + 3x_3 = 0 \quad W : x_2 - 2x_3 - x_4 = 0.$$ 

6. Let $V = \langle v_1, v_2, v_3 \rangle \subset \mathbb{R}^5$.

$$v_1 = (1,2,3,2,1) \quad v_2 = (0,1,-2,-1,0) \quad v_3 = (1,4,-1,0,1).$$

Find a basis for $V$, and a basis for $V^\perp$.

7. Give two examples (not differing only by multiplication by a constant) of $3 \times 4$ matrices $A$ with kernel given by:

$$\text{Ker}(A) = \{(1,2,-1,1), (0,1,1,2)\} \subset \mathbb{R}^4.$$ 

8. Give two examples (not differing only by multiplication by a constant) of $3 \times 4$ matrices $A$ with range given by:

$$\text{Ran}(A) = \{y = (y_1,y_2,y_3) \in \mathbb{R}^3 | y_1 + 2y_2 + y_3 = 0\}.$$
9. Find two examples (not differing only by multiplication by a constant) of $3 \times 4$ matrices $A$ with kernel as in problem 8 and range as in problem 9 above.

10. Let $v = (1, 2, 3, 4) \in \mathbb{R}^4$. Consider the subsets of the vector space $\mathbb{M}_{3 \times 4}$:

$$V = \{ A \in \mathbb{M}_{3 \times 4} | Av = 0 \};$$
$$W = \{ A \in \mathbb{M}_{3 \times 4} | \text{Ker}(A) = \langle v \rangle \}$$

Is $V$ a subspace of $\mathbb{M}_{3 \times 4}$? Is $W$ a subspace? Justify.