MATH 241 (CALCULUS III), FALL 2013-EXAM 1-9/30/2013

NAME and STUDENT ID:

**Instructions.** Use the blank sheets provided to solve the eight problems below. Turn in also the questions sheets, and don't forget to write your name on it. Closed book, closed notes, no "formula sheets", no calculators. No electronics (laptops, smartphones, etc.) allowed during the test.

**No credit for answers given without justification, even if correct. Show your work!**

There are 8 problems, each graded on a 0-10 scale, except for problems 2 and 5 (0-20 scale). *Use only the blank sheets provided-use front and back, and the back of the questions sheets, if needed.* Good luck!

**1.** Find parametric equations for the intersection of the planes

$$2x + y - 3z = 0 \quad \text{and} \quad x + y = 1.$$

Note that the point (-1,2,0) is on the line of intersection. *Hint:* Cross product of the normal vectors.

**2.** Find the velocity vector of a particle traveling to the right along the hyperbola $y = x^{-1}$ with *unit speed*, when the particle's location is $(2, \frac{1}{2})$.

**3.** For the parametrized curve $\mathbf{r}(t) = (t^2, t^3)$ find, at $t = 1$: speed $v$, unit tangent vector $\mathbf{T}$, unit normal vector $\mathbf{N}$, curvature $\kappa$.

*Hint:* Find $v$ and $\mathbf{T}$ first, then $\mathbf{N}$ (it is the unit vector perpendicular to $\mathbf{T}$, with negative first component), then $\kappa$ using $\mathbf{r}''(t) \cdot \mathbf{N} = v^2\kappa$.

**4.** Find $\mathbf{r}(t)$ and $\mathbf{v}(t)$, given the acceleration vector $\mathbf{a}(t)$ and the initial velocity and position.

$$\mathbf{a}(t) = (0, 0, \cos t), \quad \mathbf{v}(0) = (1, -1, 0), \quad \mathbf{r}(0) = (1, 0, 0).$$

**5.** Find the linearization of $f(x, y, z) = x\sqrt{y + z}$ at (8,4,5), and the direction from that point along which the function increases at the fastest possible rate. (Recall directions are defined by unit vectors.).

TURN THE PAGE FOR THREE MORE QUESTIONS.

**6.** Let $f(x,y)$ be a function of two variables, with partial derivatives $f_x, f_y$ at a point $(a,b)$. Write down the *definition* of "$f$ is differentiable at $(a,b)$".

**7.** Find a function $f(x,y,z)$ such that $\nabla f = (2z, 2y, 2x)$ (you may assume that such a function exists.)

**8.** Find an equation for the tangent plane to the surface defined by the equation below, at the point given:

$$xz + 2x^2y + y^2z^3 = 11, \quad P = (2,1,1).$$