MATH 231, FALL 2022–FINAL EXAM

NAME:

1. [12] Solve the initial-value problem. Include the largest *interval* where the solution is defined.

$$t^3 x' + 3t^2 x = t, \quad x = x(t), \quad x(-2) = 0.$$

2. [13] Sketch the phase line for $y' = (y-1)(y-2)(y-3), y = y(t)$, and classify its equilibria (as stable, unstable or semi-stable). Then sketch the y vs. t graph of solutions (include two solution curves in each range defined by the equilibria.) Indicate the vertical asymptotes where they occur.

3. [13] Solve the initial-value problem (in implicit form, if necessary.)
$(\frac{1}{x} + 2y^2x)dx + (2yx^2 - \cos y)dy = 0, \quad y = y(x), x > 0, y(1) = \pi.$

4. [12] Transform the Bernouilli-type equation below to a linear equation (for a new function $y(t)$):

$$x' + tx^3 + \frac{x}{t} = 0, \quad x = x(t), t > 0.$$

(There's no need to solve the transformed linear equation.)

5. [12] Find the general solution of the non-homogeneous problem.

$$y'' + 2y' + 2y = t.$$

6. [12] Solve the initial-value problem and sketch the graph of the solution:

$$y'' + 3y' + 2y = 0, \quad y = y(t), \quad y(0) = -1, y'(0) = 3.$$

7. [13] Find the first three non-zero terms in a power series expansion about $x = 0$ of a the solution to the initial-value problem, and state where the solution converges:

$$y'' - xy' + 2y = \cos x, \quad y = y(x), \quad y(0) = 1, y'(0) = 1.$$

Given: $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

8. [13] Use Laplace transforms to solve the initial value problem, for an arbitrary function $g(t)$ (of exponential order). Express part of the solution as an integral involving $g(t)$.

$$y'' - 2y' + y = g(t), \quad y = y(t), y(0) = -1, y'(0) = 1.$$

BRIEF TABLE OF LAPLACE TRANSFORMS

$$e^{at} ---> \frac{1}{s-a} \qquad \sin(bt) ---> \frac{b}{s^2+b^2} \qquad \cos(bt) ---> \frac{s}{s^2+b^2} \qquad t^n ---> \frac{n!}{s^{n+1}}$$

$$e^{at} f(t) ---> F(s-a) \qquad f'(t) ---> sF(s) - f(0) \qquad f''(t) ---> s^2 F(s) - sf(0) - f'(0)$$