Math 231 fall 2010- Exam 1, 9/16/2010. No credit for answers without justification. Closed books, closed notes. Calculators allowed. Time given: 75 minutes. **(Five problems, 20 pts each).**

1. An object of mass 100 kg is released from rest from a boat in the water and allowed to sink. Gravity pulls the object down, and a buoyancy force equal to 1/40 times the weight (mg) of the object pushes it up. Water resistance exerts a force on the object proportional to velocity, with proportionality constant 10 N-sec/m. Find an explicit expression for $v(t)$, the velocity of the object of a function of time, and *sketch its graph.* (You may use $g = 10m/s^2$.)

2. Solve the initial-value problem below, including the largest *interval* where the solution is defined.

$$xy' + 2y = 5x^3, \quad y = y(x), \quad y(-1) = 3.$$

3. Consider the first-order equation:

$$t + y = (t - y)y', \quad y = y(t).$$

(i) Write the equation in the form $y' = f(t, y)$, and answer (without solving the equation): for which initial conditions $y(t_0) = y_0$ is a solution guaranteed to exist?

(ii) Solve the initial-value problem defined by the equation and the condition $y(1) = 2$ (note the equation is of homogeneous type). Include the largest *interval* (containing 1) where the solution is defined.

4. (i) Show that the equation given below is *exact* and (ii) find the solution with initial condition $y(0) = \pi/2$, in implicit form.

$$e^t \sin y + e^{-y} = (te^{-y} - e^t \cos y)y', \quad y = y(t).$$

5. For the autonomous first-order equation given below: (i) find and classify the equilibria (constant solutions), as stable or unstable. (ii) Draw a diagram of all solutions, showing two solutions in each range determined by the equilibria. (iii) Indicate in your diagram which solutions have domain $\mathbb{R}$, and which are defined in an interval; in the latter case, show also the vertical asymptotes for typical solutions.

$$y' = -y(y^2 - 4), \quad y = y(x).$$