Math 231 fall 2008- Exam 3, 11/25/08 No credit for answers without justification. Closed books, closed notes, no '*formulas*' given. Calculators OK. Time given: 75 min.

1. For the second-order equation:

$$(x+1)y'' - 3xy' + 2y = 0, \quad y = y(x)$$

find the general solution, in the form:

$$y(x) = a_0 y_0(x) + a_1 y_1(x),$$

where $y_0(x)$ and $y_1(x)$ are power series at 0, with coefficients given explicitly up to $O(x^5)$. What is the interval of convergence?

2.(i) Use Laplace transforms and convolution to give a formula for the solution to the initial-value problem:

$$y'' - 2y' - 5y = g(t), \quad y = y(t), \quad y(0) = 0, y'(0) = 1,$$

where $g(t)$ is an arbitrary piecewise-continuous function for $t \geq 0$. (The answer should be given in the original variable t.)

(ii)Now let $g(t) = \begin{cases} 0, & 0 \leq t < 1 \text{ or } t \geq 2; \\ 17, & 1 \leq t < 2 \end{cases}$ Find the solution $y(t)$ of the IVP explicitly (your answer should not contain 'step functions'). Is the solution $y(t)$ continuous at $t = 1$ and $t = 2$?

3. Find a one-parameter family of solutions for the non-linear equation:

$$(y+1)y'' = 3(y')^2, \quad y = y(x).$$

(*Hint:* Let $y' = v$; then $y'' = v\frac{dv}{dy}$ leads to a first-order equation for $v(y)$.)

4. A particle of unit mass moving subject to a an attractive central force has trajectory given in polar coordinates by:

$$r = e^\theta, \quad r = r(t), \theta = \theta(t) \quad \text{(a spiral)}.$$

Find an expression for the force $\vec{F}(r) = f(r)\vec{u}_r$ (that is, find $f(r)$.) *Hint:* Use $r^2\theta' = l$, a constant of motion (angular momentum) and remember $f(r) = r'' - r(\theta')^2$.