Math 231 fall 2008- Exam 2, 10/30/08 No credit for answers without justification. Closed books, closed notes, no '*formulas*' given. Calculators OK. Time given: 75 min.

1. Find the general solution:

$$y'' - 3y' + 2y = t, \quad y = y(t).$$

2. A damped oscillator moving under an external periodic force is described by the differential equation:

$$x'' + 2x' + 16x = F\sin(\omega t), \quad x = x(t).$$

(i) Find the amplitude of the steady-state motion as a function of F and ω;

(ii) Find the value of ω (if one exists) for which this steady-state amplitude is maximized.

3. A particle of mass 1 kg moves on a line under a *repelling* force proportional to distance to the origin; the force equals 9N when the particle is at $x = 1$ m.

(i) Write down the differential equation of motion and its general solution;

(ii) For a given $x_0 > 0$, find a condition on $v_0 > 0$ so that if the particle starts at x_0 at $t = 0$, with *negative* initial velocity $x'(0) = -v_0$, it eventually goes past the origin. (The answer depends on the starting position x_0)

4. For the 2×2 linear system:

$$X' = AX, \quad X = X(t) \in \mathbb{R}^2, \quad A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

(i) Find the eigenvalues and corresponding eigenspaces;

(ii) Write down the general solution (in vector form);

(iii) Sketch a few representative solutions in the phase plane (x, y); include the eigenspaces in your diagram, and at least one solution in each region of the plane defined by them.

5. Consider the non-homogeneous system for functions $x(t), y(t)$:

$$\begin{cases} 3x' + 3x + 2y = e^t \\ 4x - 3y' + 3y = 3t \end{cases}$$

Find the general solution. (It is enough to answer with either $x(t)$ or $y(t)$; you are not required to find both.)