**Problems on Laplace transforms.** (Some are adapted from Nagle/Saff/Snyder)

**1.** Use Heaviside's formula to find the inverse transform:

$$F(s) = \frac{2s+1}{s(s-2)(s+3)}.$$

**2.** Solve the initial-value problems:

$$(i) y'' + 3ty' - 6y = 1, \quad y(0) = y'(0) = 0 \quad \text{Ans. } t^2/2$$

$$(ii) ty'' - 2y' + ty = 0, \quad y(0) = 1, y'(0) = 0 \quad \text{Ans. } \cos t + t\sin t + c(\sin t - t\cos t).$$

*Hint*: Use the convolution formula to find the inverse transform of $1/(s^2+1)^2$. Ordinarily you wouldn't expect the solution to depend on an arbitrary constant, but here this is fine. Why?

**3.** Find an inverse Laplace transform (and sketch its graph):

$$(i) \frac{e^{-5s}}{s^2} \qquad (ii) \frac{e^{-4s}}{s^2+9}$$

**4.** Solve the initial-value problem:

$$y'' + 2y' + 2y = g(t), \quad y(0) = 1, y'(0) = 0, \quad g(t) = \begin{cases} 0, t < \pi \text{ or } t > 2\pi \\ 2, \pi < t < 2\pi \end{cases}$$

**5.** Plot the functions and find their Laplace transforms:

$$(i) f(t) = \begin{cases} 1, 0 < t < 2 \\ 0, 2 < t < 4 \end{cases} \text{ ( periodic of period 4)}$$

$$(ii) f(t) = \begin{cases} \sin t, 0 < t < \pi \\ 0, \pi < t < 2\pi \end{cases} \text{ ( periodic of period } 2\pi)$$

**6.** Solve the initial-value problem:

$$y'' + 4y = g(t), \quad y(0) = 1, y'(0) = 0, \quad g(t) = \begin{cases} \sin t, 0 < t < 2\pi \\ 0, t > 2\pi \end{cases}$$

**7.** Use the convolution theorem to find(explicitly) an inverse Laplace transform:

$$(i) \frac{s}{(s^2+1)^2} \quad (ii) \frac{s+1}{(s^2+4)^2}$$

**8.** Give a formula (for arbitrary $g(t)$) for the solution of the initial-value problem:

$$y'' - 2y' + 5y = g(t) \quad y(0) = 0, y'(0) = 2.$$