Problems on Laplace transforms.  (Some are adapted from Nagle/Saff/Snyder)

1. Use Heaviside’s formula to find the inverse transform:
   \[ F(s) = \frac{2s + 1}{s(s - 2)(s + 3)} \]

2. Solve the initial-value problems:
   \[(i) y'' + 3ty' - 6y = 1, \quad y(0) = y'(0) = 0 \quad \text{Ans. } t^2/2 \]
   \[(ii) ty'' - 2y' + ty = 0, \quad y(0) = 1, y'(0) = 0 \quad \text{Ans. } \cos t + t \sin t + c(\sin t - t \cos t) \]
   \text{Hint: Use the convolution formula to find the inverse transform of } 1/(s^2 + 1)^2. \text{ Ordinarily you wouldn’t expect the solution to depend on an arbitrary constant, but here this is fine. Why?} 

3. Find an inverse Laplace transform (and sketch its graph):
   \[(i) \frac{e^{-5s}}{s^2} \quad (ii) \frac{e^{-4s}}{s^2 + 9} \]

4. Solve the initial-value problem:
   \[y'' + 2y' + 2y = g(t), \quad y(0) = 1, y'(0) = 0, \quad g(t) = \begin{cases} 0, t < \pi \text{ or } t > 2\pi \\ 2, \pi < t < 2\pi \end{cases}\]

5. Plot the functions and find their Laplace transforms:
   \[(i) f(t) = \begin{cases} 1, 0 < t < 2 \\ 0, 2 < t < 4 \end{cases} \quad \text{(periodic of period 4)}
   \[(ii) f(t) = \begin{cases} \sin t, 0 < t < \pi \\ 0, \pi < t < 2\pi \end{cases} \quad \text{(periodic of period 2\pi)}\]

6. Solve the initial-value problem:
   \[y'' + 4y = g(t), \quad y(0) = 1, y'(0) = 0, \quad g(t) = \begin{cases} \sin t, 0 < t < 2\pi \\ 0, t > 2\pi \end{cases}\]

7. Use the convolution theorem to find (explicitly) an inverse Laplace transform:
   \[(i) \frac{s}{(s^2 + 1)^2} \quad (ii) \frac{s + 1}{(s^2 + 4)^2}\]

8. Give a formula (for arbitrary \(g(t)\)) for the solution of the initial-value problem:
   \[y'' - 2y' + 5y = g(t) \quad y(0) = 0, y'(0) = 2.\]