MATH 241- EXAM 4- November 29, 2006. **NAME**:

1. *(homework-Stewart)* A fluid has density 800 kg/m^3, and flows with velocity vector field $\vec{v} = (z, y^2, x^2)$ (components in meters per second). Find the time rate of change of the total mass of fluid inside the region bounded by the cylinder $x^2 + y^2 = 4, 0 \leq z \leq 1$, in kilograms per second. (This is the same as the rate of flow outward through the boundary of the region).

2. *(homework-Stewart)* Use Gauss's law to find the charge contained in the hemisphere $x^2 + y^2 + z^2 \leq 1, z \geq 0$, if the electric field on the boundary of the region is given by $\vec{E} = (x, y, 2z)$.

3. *(handout, probl.2)* Use Stokes' theorem to show that, if C is the intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $x + y + z = 0$, we have:

$$\int_C ydx + zdy + xdz = \pi a^2 \sqrt{3}.$$

Specify how C must be oriented.

4. *(homework-Stewart)* Let C be the curve of intersection of the surfaces $z = y^2 - x^2$ and $x^2 + y^2 = 1$, oriented counterclockwise as seen from above. Let S be *any* oriented surface with boundary C, and consider the vector field:

$$\vec{F} = (ax^3 - 3xz^2, x^2y + by^3, cz^3).$$

(**i**) Explain in detail why there is a condition on $\vec{F}$ which guarantees that the value of $\int\int_S \vec{F} \cdot \vec{N} dS$ is independent of the surface S spanning C. (*Hint:* The argument uses either Stokes' theorem or the divergence theorem.)

(**ii**) For the given $\vec{F}$, find the values of a, b, c for which $\vec{F}$ satisfies the condition.

5. *(cp. hw problem 13.6.25)* Find the flux of the vector field $\vec{v} = (2y, x, -z)$ across the surface:

$$S = \{(x, y, z); z = x^2 - 2y^2, x^2 + y^2 \leq 9\}$$

(a hyperbolic paraboloid, given as a graph over a disk of radius 3), oriented by the *upward* unit normal.