MATH 241- EXAM 2- October 10, 2006. **NAME**:

1.(i)[4] Write down the second order Taylor approximation $T_2f(x,y)$ at the origin: $f(x,y)=e^x\cos y$.

(ii) [4] Give an estimate for the absolute error $|T_2f(x,y)-f(x,y)|$ valid for $|x|\leq h, |y|\leq h$, with explicit numerical constants ($0<h<1$ arbitrary).

2.[8] Find the maximum and minimum *VALUES* of $f(x,y)$ in the region $\{|x|\leq 1, |y|\leq 1\}$.

$$f(x,y)=xy^2+x^2.$$

3.[10] Find the points on the surface $x^2y^2z^8=1$ that are closest to the origin.

4.(i)[4] Classify the quadratic form in $Q(x,y,z)$ corresponding to the 3x3 symmetric matrix given below (as degenerate/nondegenerate and positive/negative definite or indefinite.) If indefinite, give the index.

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 4 \end{bmatrix}$$

(ii)[4] Sketch level sets for the quadratic form $Q(x,y,z)=c$, corresponding to $c=1$ and $c=-1$. Don't include the axes.

5.[8] Identify and parametrize the surface; you must include the domain where the parameters are defined.

$$-4x^2+y^2-2z^2-1=0, \quad y\geq 0$$

6.(i)[4]The function of two variables with level sets sketched on Fig. 1 has critical points at $(-1,0)$ (a local max) and $(0,0)$ (a saddle point). The coordinate half-axes are integral curves of its gradient vector field, and so are the two halves of the line $x=-1$. Put ARROWS on these integral curves in the direction of the flow of the gradient, and sketch one more integral curve (with arrows) in each of the six regions into which these half-lines divide the plane.

(ii) [4] Consider the vector field with integral curves sketched in fig. 2. Can this vector field be conservative? Justify.