Chapter 7 problems- answers and hints.

7.8 (a) \( u(x, y) = (\sinh x / \sinh \pi) \sin y \). (b) Use the strong maximum principle.

7.10 \( a = \pi^2 / 3 \). \( u(x, y) = \sum_{n=1}^{\infty} a_n \cos(nx) \cosh(ny) / \cosh n\pi + \text{const.} \), where \( x^2 - \frac{x^2}{3} \sim \sum_{n=1}^{\infty} a_n \cos n x \) is a Fourier cosine series on \([0, \pi]\). (\text{const.} is an arbitrary constant.)

7.12 \( u(x, y) = \frac{1}{2} (y + 1) + \sum_{n=1}^{\infty} a_n \cos(nx/2) \sinh[n(y + 1)/2] / \sinh n \), where \( \sin 2x \sim \sum_{n=1}^{\infty} a_n \cos(nx/2) \) is a Fourier cosine series on \([0, 2\pi]\).

Remark: Note that the function \( f(x) = 1 + \sin 2x \) does not satisfy the ‘compatibility condition’ \( f_x(0) = f_x(2\pi) = 0 \), so \( u_x(x, y) \) will not be continuous at the corners \((0, 1), (2\pi, 1)\). (And this cannot be ‘cured’ by adding a harmonic polynomial).

7.15 (a) Use the 2nd derivative test in 2 variables. (b) Imitate the proof of the weak maximum principle; the fact that \( x > 0 \) in \( D \) has to be used. (c) Follows from part (b) and considering the difference of 2 solutions.