Chapter 7 problems- answers and hints.

7.8 (a) $u(x,y) = (\sinh x/\sinh \pi)\sin y$. (b) Use the strong maximum principle.

7.10 $a = \pi^2/3$. $u(x,y) = \sum_{n=1}^{\infty} a_n \cos(nx)\cosh(ny)/\cosh n\pi + const.$, where $x^2 - \frac{\pi^2}{3} \sim \sum_{n=1}^{\infty} a_n \cos nx$ is a Fourier cosine series on $[0,\pi]$. (*const.* is an arbitrary constant.)

7.12 $u(x,y) = \frac{1}{2}(y+1) + \sum_{n=1}^{\infty} a_n \cos(nx/2)\sinh[n(y+1)/2]/\sinh n$, where $\sin 2x \sim \sum_{n=1}^{\infty} a_n \cos(nx/2)$ is a Fourier cosine series on $[0,2\pi]$.

Remark: Note that the function $f(x) = 1 + \sin 2x$ does not satisfy the 'compatibility condition' $f_x(0) = f_x(2\pi) = 0$, so $u_x(x,y)$ will not be continuous at the corners $(0,1)$, $(2\pi,1)$. (And this cannot be 'cured' by adding a harmonic polynomial).

7.15 (a) Use the 2nd derivative test in 2 variables. (b) Imitate the proof of the weak maximum principle; the fact that $x > 0$ in D has to be used. (c) Follows from part (b) and considering the difference of 2 solutions.