Math 400, Test 3, 7/7/2006- PART 2 (open book/notes)

1. Riemann's (1854) Habilitation thesis on representation of functions by their Fourier series was influenced by Dirichlet's theorem (1829). State this theorem, and sketch the graphs of two *discontinuous* periodic functions- one satisfying the theorem's hypotheses, the other violating it.

2. Consider the sequence of continuous functions, defined in the interval $[0,1]$: $f_n(x) = x^n$, $n \geq 1$. Find the pointwise limit f of the f_n in $[0,1]$; explain (based on the definition or on a geometric characterization) why the convergence is not uniform in $[0,1]$.

3. State the main theorem in Riemann's 1859 paper on the distribution of prime numbers (just the two largest terms). Explain (in outline) how this formula implies the Prime Number Theorem $\pi(x) \sim \int_2^x \frac{dt}{\log t}$.

4.[2pts] State *(precisely)* how the following three basic questions were resolved, and *by whom.*

1) Characterize the dense subsets of $\mathbb{R}$ which can be sets of continuity of a function;

2) How discontinuous can an integrable function be?

3) What can be said about the set of continuity of the derivative f', if f is differentiable in an interval (a,b)? Do one-sided limits exist at a discontinuity?

5. True or false? If true, who proved it? If false, how can a counterexample be obtained?

(a) The complement of a meager set is dense;

(b) There exist uncountable meager sets;

(c) The set of algebraic numbers in $[0,1]$ has measure zero.