1. Riemann’s (1854) Habilitation thesis on representation of functions by their Fourier series was influenced by Dirichlet’s theorem (1829). State this theorem, and sketch the graphs of two discontinuous periodic functions—one satisfying the theorem’s hypotheses, the other violating it.

2. Consider the sequence of continuous functions, defined in the interval $[0, 1]$: $f_n(x) = x^n$, $n \geq 1$. Find the pointwise limit $f$ of the $f_n$ in $[0, 1]$; explain (based on the definition or on a geometric characterization) why the convergence is not uniform in $[0, 1]$.

3. State the main theorem in Riemann’s 1859 paper on the distribution of prime numbers (just the two largest terms). Explain (in outline) how this formula implies the Prime Number Theorem $\pi(x) \sim \int_2^x \frac{dt}{\log t}$.

4. [2pts] State (precisely) how the following three basic questions were resolved, and by whom.

   1) Characterize the dense subsets of $\mathbb{R}$ which can be sets of continuity of a function;
   2) How discontinuous can an integrable function be?
   3) What can be said about the set of continuity of the derivative $f'$, if $f$ is differentiable in an interval $(a, b)$? Do one-sided limits exist at a discontinuity?

5. True or false? If true, who proved it? If false, how can a counterexample be obtained?

   (a) The complement of a meager set is dense;
   (b) There exist uncountable meager sets;
   (c) The set of algebraic numbers in $[0, 1]$ has measure zero.