The Prime Number Theorem - Riemann's 1859 paper
(sections in 'Prime Obsession', by J.Derbyshire)
Ideally you should read the whole book (it’s fun!), but the core sections dealing with
Riemann’s paper on the PNT are given below:

7.VIII- Definition of Li(x); the PNT in terms of Li; graphs (p.115/116)

Ch. 8 (historical): Riemann/ Chebyshev and his results (1849, 1850)/ Dedekind and
Riemann in Göttingen- mid 1850s

Ch. 9: idea of ‘domain stretching’/ graphs of the extended zeta function/ the ‘functional
relation’ for zeta

Ch. 10 (historical): landmarks on the way to the PNT, 1859-1896

Ch. 14 (historical): Contributions to RH by Bohr and Landau/ Hardy and Littlewood

Ch. 15: Orders of growth/ the Möbius function/ Thomas Stieltjes’ claim (1885)

Ch. 19: the functions π(x) and J(x)- graphs/ Möbius inversion: π(x) from J(x)/ integral
formulation of the Euler product formula (“Golden Key”)

Ch. 21: Riemann’s formula for J(x) (expression 21-1)/ calculation of π(1,000,000) from
the formula (Table 21-1); discussion of the contribution from the ‘oscillatory terms’,
coming from zeta zeros.

Other sources: the first source below contains complete accounts of Riemann’s paper
(including a translation) and the proofs of the PNT by Hadamard and de la Vallée
Poussin, and discussion of many related issues. The second source is an introduction to
basic techniques in analytic number theory, centered on the PNT (but with minimal
prerequisites); it includes a proof of Dirichlet’s theorem on primes in arithmetic
progressions. Both references assume the reader knows basic facts in complex analysis
(Math 443 here at U.T.K.- offered this fall!)

1-H.M Edwards, Riemann’s Zeta Function (Dover, 2001)
texts 53 (2003)