Math 241, Spring 2006 (Alex Freire)

LIST 1: **Parametrized curves, motion in space, vector-valued functions of one variable**

motion in the plane

1. $r(t)$ is the position of the particle in the xy plane at time t. Find an equation in x and y whose graph is the path of the particle; then find the velocity and acceleration vectors at the given t.

$$r(t) = (t + 1, t^2 - 1), \quad t = 1$$

motion in space

2. $r(t)$ is the position of the particle at time t. Find the velocity and acceleration vectors; then write the velocity vector at the given time as the product of speed and direction vector.

$$r(t) = (t, \cos 2t, \sin 2t), \quad t = 1$$

tangent line

3. For the curve in problem **2**, find parametric equations for the tangent line to the curve at the given t.

integration of vector-valued functions

4. Given the velocity vector at time t and the position vector at time $t = 0$, find the position vector $r(t)$ for arbitrary t.

$$v(t) = (t, -t^2, 2t); \quad r(0) = (1, 2, 3)$$

arc length, unit tangent vectors

5. For the given curve, find (i) the arc length parameter s; (ii) the unit tangent vector; (iii) the length of the given arc of the curve

$$r(t) = (4 \cos t, 3t, 4 \sin t), \quad t \in [0, 2\pi]$$

independence of parametrization

6. Calculate the length of one turn of the helix $r(t) = (4 \cos t, 4 \sin t, 3t), t \in [0, 2\pi]$, in the given parametrization and in the alternative parametrization:

$$r(t) = (4 \cos 3t, 4 \sin 3t, 9t), t \in [0, 2\pi/3]$$

second-order Taylor expansion

7. For the parametrized curve in problem **2.**, use the velocity and acceleration vectors to find the degree two Taylor polynomial of $r(t)$ at the given value of t.

curvature in the plane

8. Find the unit tangent and unit normal vectors and the curvature of the parametrized curve (and sketch the curve):

$$r(t) = (t, (1/3)t^3), t \in [-1, 1]$$

curvature in space

9. Find the unit tangent, unit normal and curvature for the space curve:

$$r(t) = (\cosh t, t, -\sinh t)$$

motion on a curve with constant speed

10. A particle is moving on the given curve with constant speed v (say $v = 10$) Given its position at time $t = 1$), find the velocity and acceleration vectors at $t = 1$ (assume the velocity vector has negative x component).

$$\frac{x^2}{4} + \frac{y^2}{9} = 1, \quad r(1) = (0, 3)$$

tangential and normal components of the acceleration vector

11. Find the tangential and normal components of the acceleration vector at the given value of t (that is, find numbers a_T, a_N and unit tangent (resp. normal) vectors T (resp. (N) so that $a(t) = a_T T + a_N N$.)

$$r(t) = (t^2, 2t, t + 1), \quad t = 1$$

mean-value inequality

12. Given the velocity vector for the motion, estimate the distance $||r(2) - r(0)||$:

$$v(t) = (e^t \sin t, e^t \cos t, \cos^3(2t))$$