Math 241, section 7- EXAM 4- May 8, 2006, 5:00-7:00, Ayres 314. Closed book, closed notes. No calculators. Show all work.

1.(i)[5] Compute the work done by the vector field $\mathbf{F}$ in moving a particle along the closed oriented path given.

$\mathbf{F}(x,y,z) = (y, x, y+1)$; C: triangle with vertices $(0,0,0), (1,1,1), (-1,1,1)$ (in this order).

(ii)[4] Is the vector field $\mathbf{F}$ conservative? Justify.

2. Find a potential function for each of the following conservative vector fields (no points for checking it is conservative- that's given.)

(i)[4] $\mathbf{F}(x,y,z) = (4xy - 3x^2z^2, 2x^2, -2x^3z - 3z^2)$.

(ii)[4] $\mathbf{F}(r) = \frac{-1}{r^5}\mathbf{u}_r$, where $\mathbf{u}_r$ is the unit radial vector field in $\mathbb{R}^3 - \{0\}$. (Recall the gradient of a function $u(x,y,z) = f(r)$ depending only on the distance r to the origin is $\nabla u = f'(r)\mathbf{u}_r$.)

3. (i)[4] Find z_x and z_y when $(x,y,z) = (2,3,6)$, if $z(x,y)$ is defined implicitly near this point by the relation $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

(ii) [4] Verify that $u(x,t) = \sin(x+3t) + x + 3t$ is a solution of the one-dimensional wave equation with speed 3: $u_{tt} = 9u_{xx}$.

4. Let S be the surface $x^2+y^2+z^2 = 4, z \geq 0$ (hemisphere) oriented by the upward unit normal, $\mathbf{F}$=$(x^2e^{yz}, y^2e^{xz}, z^2e^{xy})$.

(i)[4] Compute curl $\mathbf{F}$ and find a parametrization of the boundary of S, with the correct orientation.

(ii)[6] Use Stokes' theorem to compute the flux of curl $\mathbf{F}$ over the surface S (with the upward unit normal).

5.[5] Can there be a vector field $\mathbf{G}$ in $\mathbb{R}^3$ so that

$$curl\mathbf{G} = (x(e^y - e^z), -e^y, e^z - e^y)?$$

Justify.

6.(i)[4] Let T be the upper solid hemisphere $\{x^2+y^2+z^2 \leq 1, z \geq 0\}$ and $\mathbf{F} = (x^2 + yz^5, x^3 + y^2 + \sin z, x^2 + y^2)$. Show that the integral of div $\mathbf{F}$ over T is zero.

(ii)[6] The flux of $\mathbf{F}$ over the upper unit hemisphere $x^2+y^2+z^2 = 1, z \geq 0$ (outward normal) is equal to the flux of $\mathbf{F}$ over the unit disk $x^2+y^2 = 1, z = 0$ (upward normal). Justify this based on (i) and the divergence theorem, and compute this flux.