1. (i) [5] Compute the work done by the vector field $F$ in moving a particle along the closed oriented path given.

$F(x, y, z) = (y, x, y + 1)$; $C$: triangle with vertices $(0, 0, 0), (1, 1, 1), (-1, 1, 1)$ (in this order).


2. Find a potential function for each of the following conservative vector fields (no points for checking it is conservative- that’s given.)

(i) [4] $F(x, y, z) = (4xy - 3x^2z^2, 2x^2, -2x^3z - 3z^2)$.

(ii) [4] $F(r) = -r^{-1}u_r$, where $u_r$ is the unit radial vector field in $\mathbb{R}^3 - \{0\}$.

(Recall the gradient of a function $u(x, y) = f(r)$ depending only on the distance $r$ to the origin is $\nabla u = f'(r)u_r$.)

3. (i) [4] Find $z_x$ and $z_y$ when $(x, y, z) = (2, 3, 6)$, if $z(x, y)$ is defined implicitly near this point by the relation $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

(ii) [4] Verify that $u(x, t) = \sin(x + 3t) + x + 3t$ is a solution of the one-dimensional wave equation with speed 3: $u_{tt} = 9u_{xx}$.

4. Let $S$ be the surface $x^2 + y^2 + z^2 = 4, z \geq 0$ (hemisphere) oriented by the upward unit normal, $F = (x^2e^{yz}, y^2e^{xz}, z^2e^{xy})$.

(i) [4] Compute curl $F$ and find a parametrization of the boundary of $S$, with the correct orientation.

(ii) [6] Use Stokes’ theorem to compute the flux of curl $F$ over the surface $S$ (with the upward unit normal).

5. [5] Can there be a vector field $G$ in $\mathbb{R}^3$ so that

\[
\text{curl } G = (x(e^y - e^z), -e^y, e^z - e^y)\]

Justify.

6. (i) [4] Let $T$ be the upper solid hemisphere $\{x^2 + y^2 + z^2 \leq 1, z \geq 0\}$ and $F = (x^2 + y^2, x^3 + y^2 + \sin z, x^2 + y^2)$. Show that the integral of div $F$ over $T$ is zero.

(ii) [6] The flux of $F$ over the upper unit hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$ (outward normal) is equal to the flux of $F$ over the unit disk $x^2 + y^2 = 1, z = 0$ (upward normal). Justify this based on (i) and the divergence theorem, and compute this flux.