Math 241, Spring 2006 Exam 2 (3/14/2006), NAME:

Instructions. Closed book, closed notes, no calculators. No credit for answers without justification—show all work. 50-min test. 10 points per problem.

1. (List 4-12) (i) Identify and sketch the surface given below:

\[-4x^2 + y^2 + z^2 = 4, \quad x \geq 0.\]

(ii) This surface may be parametrized by a map \(F : D \rightarrow \mathbb{R}^3\) of the form:

\[F(u, v) = (\sinh v, A \cosh v \cos u, A \cosh v \sin u).\]

Find the constant \(A\) and the domain \(D\) in the \((u, v)\) plane (use \(\cosh^2 v - \sinh^2 v = 1\) for all \(v \in \mathbb{R}\)).

2. (List 4-4) Find the absolute maximum and minimum VALUES of \(f(x, y)\) in the region \(D\).

\[f(x, y) = 4y - y^2 - 2xy + x^2, \quad D = \{(x, y); 0 \leq x \leq 1, 0 \leq y \leq 1\}.\]

Hint: \(f\) is increasing or decreasing along each side of the square \(D\), so you only need to check the interior and the four vertices.

3. (List 4-6) Find the largest possible AREA for a rectangle with sides parallel to the coordinate axes, and vertices on the ellipse \(x^2 + 4y^2 = 4\).

Outline: The area is 4xy (for \(x > 0, y > 0\)). Set up the Lagrange multiplier system; multiply one equation by \(x\), the other by \(y\), add them together and use the constraint.

4. (List 5-4) Find the volume of the solid region bounded by the cylinder \(x^2 + y^2 = 9\) and the planes \(z = 0, z = y + 5\) (note that the function \(f(x, y) = y + 5\) is positive on the disk \(x^2 + y^2 \leq 9\), so the volume is given by a double integral, easily computed using polar coordinates.)

5. (List 5-8) Find the area of the surface (one-half of a paraboloid):

\[y = x^2 + z^2, \quad z \geq 0, \quad y \leq 9.\]

(you may use \(x = v \cos u, z = v \sin u, y = v^2\), for appropriate ranges of \(u\) and \(v\).)