Math 241, Spring 2006- EXAM 1. NAME:

**Instructions.** Closed book, closed notes, no calculators. No credit for answers without justification- show all work. 50-min test.

1. (*List 1:3,5*) [10] For the parameterized helix \( r(t) = (12t, 5 \sin t, 5 \cos t), t \in [0, 2\pi] \), find: (i) the total length; (ii) parametric equations for the tangent line at the point \((4\pi, 5\sqrt{3}/2, 5/2)\)

2. (*List 1:10*) [10] A particle moves with constant speed \( v = 3 \) on the ellipse \( x^2 + 2y^2 = 12 \) (counterclockwise). When the particle is at the point \((2, -2)\), find: (i) the velocity vector; (ii) the direction of the acceleration vector (recall directions are given by unit vectors).

3. (*List 3:8*) [5] Let \( f(x, y) = x^3 + y^3 + 6xy \). The origin is a critical point. Let \( r(t) \) be a smooth curve satisfying \( r(0) = (0, 0), r'(0) = (1, 1) \). Does the function of one variable \( g(t) = f(r(t)) \) have a local max, local min or an inflection point when \( t = 0 \)?

4. (*List 3:5*) [5] Find a unit normal vector to the hyperboloid \( 2x^2 + 2y^2 - 3z^2 = 1 \), at the point \((1, 1, 1)\).

5. (*List 2:8*) [5] Let \( f(x, y, z) = x^{-1}y^5z^3 \) \((x \neq 0)\): assume the values of \( x, y, z \) are known up to an error, so that \(|\Delta x|/x| \leq 0.005\), with the same relative error estimates for \( y \) and \( z \). Estimate the relative error \(|\Delta f|/f\) in the computed value of \( f \).

6. (*List 3:7,14*) [10] Let \( f(x, y) = x^4 + y^4 + 4xy \). (i) Locate a saddle point of \( f \) (if there is one); (ii) Write down the degree 2 Taylor polynomial of \( f \) at \((-1,-1)\).

7. (*Lecture: 2/10*) [5] Explain why the function in problem 6 is guaranteed to have an absolute (global) minimum, over all \((x, y) \in \mathbb{R}^2\).