1. [7] Show that the line integral below is independent of path and compute its value \((C)\) is any path from \((-1,0)\) to \((5,1)\):

\[
\int_C 2x \sin y \, dx + (x^2 \cos y - 3y^2) \, dy.
\]

2. [6] Use Green’s theorem to compute the line integral:

\[
\int_C (x^3 - y^3) \, dx + (x^3 + y^3) \, dy,
\]

where \(C\) is the oriented boundary of the region between the curves \(x^2 + y^2 = 1, x^2 + y^2 = 9\).

3. [7] Find the flux of the vector field \(\mathbf{F} = (xy, yz, xz)\) across the surface \(S\), described as follows: \(S\) is the part of the paraboloid \(z = 4 - x^2 - y^2\) above the square \(0 \leq x \leq 1, 0 \leq y \leq 1\), with the \textit{upward} orientation.

4. [6] Determine whether or not the following vector field in \(\mathbb{R}^3\) is conservative:

\[
\mathbf{F} = (2xy, x^2 + 2yz, y^2).
\]

5. [6] What is an ‘orientable surface’? Is a surface given by an equation \(F(x,y,z) = 0\) always orientable? Why?

6. [6] A thin wire is bent into the shape of a semicircle \(x^2 + y^2 = 4, y \geq 0\). If the linear mass density is constant \((=1)\) and the total mass is \(2\pi\), find the coordinates of the center of mass of the wire.

7. [6] Use Green’s theorem to find the \textit{area} of the region enclosed by the hypocycloid, a simple closed curve parametrized by \(r(t) = (\cos^3 t, \sin^3 t), 0 \leq t \leq \pi\).