1. [4] Let \( f(x, y) = x^2y^2 \), \( x(s, t) = e^s \cos t \), \( y(s, t) = e^s \sin t \). Compute the gradient vector \( \nabla u = (u_s, u_t) \) of the composition:
\[
u(s, t) = f(x(s, t), y(s, t)).
\]

2. [5,5,3,3] For the function of two variables
\[
f(x, y) = 3x - x^3 - 2y^2 + y^4:
\]
(a) Find all the critical points of \( f \);
(b) Classify the critical points \((x, y)\) with \( y > 0\) (as local maxima, local minima or saddle points);
(c) Choose one of the critical points found in (b) and write the second order Taylor approximation of \( f(x, y) \) near that point;
(d) For each critical point in part (b), sketch the approximate diagram of level sets of \( f \), in a neighborhood of that critical point.

3. [10] Find the absolute maximum and minimum values of the function of three variables:
\[
f(x, y, z) = xy + z^2
\]
under the constraint \( x^2 + y^2 + z^2 = 4 \).

4. [4,4] Consider the double integral:
\[
\int_0^1 \int_{x^2}^1 x^2 \sin(y^3) dy \, dx.
\]
(i) Sketch the region of integration;
(ii) Compute the value of the integral (change the order of integration, if necessary).