MATH 231, SPRING 2008: Homework set 4.

In the following problems, find **one** solution of the non-homogeneous second-order equation and answer the question. Use complex numbers when appropriate.

With $L[y] = 4y'' + 12y' + 13y, \quad y = y(t)$:

1. $L[y] = t^2$;

2. $L[y] = e^{-\frac{3}{2}t}\sin t$. Are there bounded solutions?

3. $L[y] = e^{-\frac{3}{2}t}$. Are there bounded solutions?

4. $L[y] = \cos 2t$. Are there periodic solutions?

Now use the operator $L[y] = y'' - 4y$. Recall that the solutions of $L[y] = 0$ are unbounded for $t > 0$, except in the special case when $y'(0) = -2y(0)$ (what happens then?)

5. $L[y] = e^{-2t}$. Are there bounded solutions?

6. $L[y] = \sin 2t$. Are there periodic solutions?

7. Consider the family of non-homogeneous equations:

$$y'' + by' + 4y = \sin 2t, \quad 0 < b < 4.$$

Show there is a periodic solution, and find its amplitude as a function of b. Give a physical interpretation. [In contrast, note that the corresponding homogeneous equation is 'underdamped' (oscillatory with decreasing amplitude), while when $b = 0$ the non-homogeneous system is "at resonance" (unbounded oscillations)]

8. A particle moves in accordance with the law:

$$y'' + 4y' + 16y = f(t), \quad y = y(t),$$

with $f(t)$ periodic, of the form $f(t) = A\sin(\omega t)$. (1) What frequency ω_f of the $f(t)$ will make the period of the steady-state motion equal to $\pi/3$? (2) What frequency ω_f of $f(t)$ will produce resonance (i.e., maximum amplification factor)? (3) What is the value of the amplification factor at resonance?

Remark: The 'amplification factor'is the ratio M/A, where M is the amplitude of the steady-state solution and A the amplitude of $f(t)$.

9. Write down a second-order equation to model the following: a particle of unit mass moves on a line under a repelling force equal to twice the distance to the origin (no friction); in addition, there is an external periodic force. Question: is the steady-state solution periodic?