Math 231, Spring 2008- Homework set 3

For problems 1-5 below: (i) solve the initial-value problems given ($y = y(t)$), then (ii) plot the graph (y vs. t) of the solutions and (iii) sketch the corresponding path (y, y') in the (y, v) plane. (iv) For problems 1 and 2, write down the conserved energy $E(y, v)$ and include in part (iii) the level set of E (an ellipse or an arc of a hyperbola) corresponding to the solution of the IVP.

1. $y'' + 4y = 0, \quad y(0) = 1, y'(0) = 1.$

2a. $y'' - 9y = 0, \quad y(0) = 1, y'(0) = 1.$
2.b $y'' - 9y = 0, \quad y(0) = 1, y'(0) = -1.$

3. $y'' + y' + y = 0, \quad y(0) = 1, y'(0) = 1.$

4. $y'' + 3y' + y = 0, \quad y(0) = 0, y'(0) = 1.$

5a. $y'' + 3y' - 4y = 0, \quad y(0) = 1, y'(0) = 0.$
5b. $y'' + 3y' - 4y = 0, \quad y(0) = 0, y'(0) = 1.$

6. Plot the coefficients of each of the equations in 1-5 above in the (b, c) plane (where the equation is $y'' + by' + cy = 0$), including in your sketch the parabola $c = b^2/4$.

7. A particle executes simple harmonic motion. A $t = 0$, it is $-5m$ from the equilibrium position, its velocity is $6m/s$ and the acceleration is $10m/s^2$. Find the equation $y(t)$ for position (in amplitude-phase form, using sines.)

8. A particle of mass 2kg moving in a straight line is *repelled* from the origin by a force proportional to the distance of the particle from 0. When the distance is $2m$, the force is 4N. Find the equation for the position of the particle as a function of time, assuming it is located at distance 1 from the origin, with zero velocity, at $t = 0$.

9. A particle moves on a straight line in accordance with the equation:

$$y'' + 10y' + 16y = 0, \quad y = y(t).$$

At $t = 0$, $y = 1$ and $y' = 0$. (1) Find the solution $y(t)$. (2) When does y attain its maximum value? Does the particle reach $y = 0$ for $t > 0$? (3) Draw a graph of the motion (y vs. t). Is the motion oscillatory?

10. A particle of mass m moves on a straight line. It is attracted toward the origin by a force equal to k times its distance from the origin. The resistance is $2R$ times the velocity. Find the maximum value of m so that the motion will *not* be oscillatory.