MATH 231, SPRING 2008: First-order linear equations.

Find the general solution; if an initial condition is given, find also that particular solution (including its interval of definition.). Include a sketch of two solutions (for each equation).

1. $y' + 2y = 3e^{-2t}, \quad y = y(t).$

2. $y' + 2y = \sin x, \quad y = y(x).$

3. $y' - y = e^x, \quad y = y(x), \quad y(0) = 1.$

4. $y' - \frac{1}{t}y = t, \quad y = y(t), \quad y(1) = 1.$

5. Compute the indefinite integral below using complex numbers:

$$\int e^{3s} \cos(4s) ds.$$

6. Write the function: $y(t) = 2\sin 3t + 4\cos 3t$ in 'amplitude-phase form': $y(t) = A\sin(\omega t + \varphi)$ (that is, find A, ω and φ.)

For each of the problems below, find the appropriate differential equation (from first principles) and its solution (including a graph), then answer the question.

7. A tank initially contains 200 l of fresh water. Salt solution of unknown concentration is poured into the tank at a rate of 10 l/min, and the mixture flows out at the same rate. At the end of 120 min, the concentration of salt in the outgoing solution is found to be 1.2 kg/l. Find the concentration of the entering solution.

8. The CO_2 content of air in a 9000 m^3 room is 0.3 percent (in volume). Fresh air containing 0.1 percent CO_2 is pumped into the room at the rate of 1000 m^3/min (and air leaves the room at the same rate). When will the CO_2 content of the air in the room be 0.2 percent?

9. The temperature in a room is 21 degrees Celsius. A thermometer which has been kept in it is placed outside. After 5 min the thermometer reading is 16 Celsius. Five minutes later, it is 13 Celsius. Find the outside temperature.

10. By natural increase (births minus deaths), a city whose present population is 40,000 would double in 50 years. There is also a net addition of 400 persons per year, because of people leaving and moving into the city. Estimate is population in 10 years. (Assume an exponential model, which is reasonable over short periods.)