MATH 231, EXAM 2-November 13, 2007.

Instructions: Closed book, closed notes. Approved calculators allowed. No credit for answers given without justification; show all work. 10 pts per problem (5 per item), except where stated otherwise. Time given: 75 min (12:40 to 1:55).

1. For $y = y(t)$, let L be the operator $L[y] = y'' - 2y' + 5y$.

(i) Solve the initial-value problem:

$$L[y] = 0, \quad y(0) = 1, \quad y'(0) = 3.$$

(ii) Find the general solution of the non-homogeneous equation:

$$L[y] = e^t + t.$$

2.[12pts.] A particle of mass 1 moves on a line, subject to a force:

$$f(y) = 8y - 4y^3.$$

(i)[4] Find the potential $U(y)$, and write down a conserved quantity $E(y, y')$ for the motion $y = y(t)$.

(ii)[4] Sketch a graph of the potential; find and classify the equilibria (as stable or unstable).

(iii)[4]If $y(0) = 1.5$ and $y'(0) = 0$, find the range of the motion. (That is, find the interval to which the particle's motion is confined.)

3. A damped harmonic oscillator is described by the differential equation of motion:

$$y'' + 2y' + 5y = 0, \quad y = y(t).$$

(i) If the initial conditions are $y(0) = 1$, $y'(0) = 1$, find the solution $y(t)$;

(ii) Find the first two time moments $0 < t_1 < t_2$ for which the solution of part (i) satisfies $y(t_1) = y(t_2) = 0$.

4.[8pts.] Find by power series methods two linearly independent solutions of:

$$y'' + \frac{t}{1-t^2}y' - \frac{1}{1-t^2}y = 0, \quad y = y(t).$$

Include at least *three nonzero terms* for each solution (unless it is a polynomial of degree less than 3), as well as the *interval of convergence* for each.

5. A weight of 200 g is attached to a pendulum of length L swinging in a medium which offers resistance equal to 2 times the *linear* velocity.

(i) Write down the differential equation of motion for $y = y(t)$, the angular deviation from equilibrium in radians (consider small oscillations only).

Hint: Recall that the equation of motion for small oscillations of the *undamped* pendulum is $mLy'' = -mgy$, and that the linear velocity is Ly'.

(ii) Find the value of L for which the period of damped oscillations equals 2π seconds (use $g = 10m/s^2$ if necessary.)