

MATH 231- EXAM 1- September 27, 2007

Instructions. Solve the following problems. For credit, show all work, not just the answer. Approved calculators allowed. 1 point per item (total: 10 points.) Time given: 75 min (12:40-13:55).

1. Find the general solution:

(i) $y' - y = e^{2t}$, $y = y(t)$;

(ii) $y' + \frac{y}{t} = \frac{y^2}{t}$, $y = y(t)$, $t \neq 0$.

2. The radium in a sample of lead decays radioactively at a rate proportional to the amount present. If 10% of the radium decays in 200 years (so that 90% of the original amount has not yet decayed), what percentage of the original amount of radium will be left in the sample after 1,000 years? What is the half-life of this radioactive decay?

3. (i) Show that the equation given below is *exact* and (ii) find the solution with initial condition $y(0) = 0$, in implicit form.

$$(t \sin y - y^2)y' - \cos y = 0, \quad y = y(t).$$

4. Consider the first-order equation:

$$t^2 y' = ty - y^2, \quad y = y(t).$$

(i) Write the equation in the form $y' = f(t, y)$, and answer (without solving the equation): for which initial conditions $y(t_0) = y_0$ is a solution guaranteed to exist?

(ii) Solve the initial-value problem defined by the equation and the condition $y(1) = 1$ (note the equation is of homogeneous type, as well as Bernoulli.). Include the *interval* where the solution is defined.

5. For the autonomous equation of Riccati type:

$$y' - y^2 + 9 = 0, \quad y = y(t)$$

(i) Find and classify the equilibria (as stable or unstable);

(ii) Solve the initial-value problem defined by the equation and the condition $y(1) = 0$. (*Hint:* Recall $3 \tanh(3t)$ and $3 \coth(3t)$ are solutions, and that $\tanh(0) = 0$.)