Supplementary problems on first-order equations.

Show that each of the following equations is exact and find the general solution, or the solution of the initial-value problem (in implicit form):

1. $\frac{2ty+1}{y} + \frac{y-t}{t^2}y' = 0, \quad y = y(t);$

2. $e^t \sin y + e^{-y} = (te^{-y} - e^t \cos y)y', \quad y = y(t) \quad y(0) = \pi/2$

Multiply the differential equation by the given integrating factor to turn it into an exact one, then find a conserved quantity for the equation.

3. $t^2 + y^2 + t = -tyy' \quad y = y(t)$, integrating factor t.

4. $y't(2t - y^3) = y(2t + y^3), \quad y = y(t)$, integrating factor $1/y^3$

Solve the following initial-value problems of Bernouilli type, including the interval where the solution is defined:

5. $y' + y = ty^3, \quad y(t) = t, \quad y(0) = 1/\sqrt{2}.$

6. $y' = -\frac{y}{t} - y^2, \quad y = y(t), \quad y(1) = 1.$

Find the general solution ('homogeneous type').

7. $2ty + (t^2 + y^2)y' = 0, \quad y = y(t)$ **8.**$(t + y) = (t - y)y', \quad y = y(t).$

Solve the initial-value problem, or find the general solution:

9. $ty' + 2y = 5t^3, \quad y = y(t)$ **10.**$(t^2 + 1)y' + ty = t, \quad y = y(t)$

11. $y' - \frac{y}{t} = te^t, \quad y(1) = e - 1$ **12.**$t^3y' + 3t^2y = t, \quad y(2) = 0$

13. $y' = \frac{1-t^2}{y^2}, \quad y = y(t)$ **14.**$y \ln t = ty', \quad y = y(t)$

15. $\sqrt{2ty}y' = 1, \quad y = y(t)$

16. $y' - y^2 + 25 = 0, \quad y(1) = 0$ (For this problem, in addition, sketch the diagram of all solutions in the (t, y) plane).

17-20 For the equations in the problems listed below, use the existence-uniqueness theorem to answer the following question: for which initial-value problems $y(t_0) = y_0$ is a unique solution guaranteed to exist, even without solving the equation explicitly? Problems: **15, 14, 12, 8**