

### Homework problems- variation of constants and mechanics.

Use 'variation of constants' to find the general solution of the following non-homogeneous equations; two solutions of the associated homogeneous equation are given:

1.  $t^2y'' - ty' + y = t$ ;  $y_1(t) = t$ ,  $y_2(t) = t \ln t$ . *Ans.*  $y_p(t) = \frac{t}{2}(\ln t)^2$

2.  $2t^2y'' + 3ty' - y = t^{-1}$ ;  $y_1(t) = t^{1/2}$ ,  $y_2(t) = t^{-1}$  *Ans.*  $y_p(t) = -\frac{1}{3}t^{-1} \ln t$

3. A particle moves on a straight line according to the equation of motion:

$$y'' + 4y' + 13y = 0, \quad y = y(t).$$

At  $t = 0$ ,  $y = 0$  and  $y' = 12$ . (1) Find the solution  $y = y(t)$ . (2) Find the time required for the damped amplitude to decrease by 50%.

4. A particle moves on a straight line in accordance with the equation:

$$y'' + 10y' + 16y = 0, \quad y = y(t).$$

At  $t = 0$ ,  $y = 1$  and  $y' = 0$ . (1) Find the solution  $y(t)$ . (2) When does  $y$  attain its maximum value? Does the particle reach  $y = 0$  for  $t > 0$ ? (3) Draw a graph of the motion ( $y$  vs.  $t$ ). Is the motion oscillatory?

5. A particle of mass  $m$  moves on a straight line. It is attracted toward the origin by a force equal to  $k$  times its distance from the origin. The resistance is  $2R$  times the velocity. Find the maximum value of  $m$  so that the motion will *not* be oscillatory.

6. A 10 kg body stretches a spring 5 cm. After it is brought to rest, it is displaced a further 6 cm, attached to a driving force  $f(t) = \sin(4\sqrt{6}t)$  N and given a downward velocity of  $4m/s$ . Find the equation of motion  $y(t)$ , where  $y$  is displacement from equilibrium. Is the motion bounded or unbounded? What is the undamped resonant frequency?

7. A particle weighing 16 lb moving on a horizontal line is attracted to the origin 0 by a force proportional to the distance to 0. When the particle is at  $y = -2$ , this force is 9 lb. In addition, a forcing function  $f(t) = \sin 3tA$  is impressed on the system. If at  $t = 0$ ,  $y = 2$  and  $y' = 0$ , find (1) the equation of motion  $y(t)$  and (2) the resonant frequency of the system.

8. A particle moves in accordance with the law:

$$y'' + 4y' + 16y = f(t), \quad y = y(t).$$

(1) What frequency  $\omega_f$  of the function  $f(t)$  will make the period of the steady-state motion equal to  $\pi/3$ ? (2) What frequency  $\omega_f$  of  $f(t)$  will produce resonance (i.e., maximum amplification factor)? (3) What is the value of the amplification factor at resonance?

**9.** A 16 lb weight stretches a spring 6 in. Its coefficient of damping is 2. The 16lb weight is removed, replaced by a 64 lb weight and brought to rest. At  $t = 0$ , a forcing function  $8 \sin(t)$  is applied to the system. Find the steady-state motion and the amplification factor of the system.