EXPONENTIAL PROCESSES- SOLVED EXAMPLES. (Math 231/Freire/ 8-28-2007)

Example 1. *(Radiocarbon dating)* Charcoal from the occupation level of the Lascaux Cave in France gave (in 1950) an average count of 0.97 disintegrations per minute per gram. Living wood gave 6.68 dpm/g. Estimate the date of the occupation (and hence of the famous paintings in the cave.) [*Braun, p.19]*

Example 2. *(Radioactive decay)* The half-life of uranium-238 is 4.5 Gyr, that of U-235 is 0.707 Gyr. In 1946, the ratio of U-238 to U-235 in any sample was 137.8. Assuming that equal amounts of U-235 and U-238 were present in any sample at the time of uranium synthesis, estimate the age of uranium. *[Braun p.18]*

Example 3. *(dilution problem)* A tank contains 400 l of water. In error, 200 kg. of salt are poured into the tank (instead of 100 kg.) To correct, the solution is allowed to flow out through a hole in the bottom of the tank, at a rate of 10 l/min. At the same time, 10l/min of fresh water are pumped into the tank. The mixture is kept uniform (by stirring.) When will it contain the desired amount of salt? *[Tenenbaum-Pollard, p.123]*

Example 4. *(temperature)* A body whose temperature is 180° is immersed in a liquid whose temperature is kept constant at 60°. In one minute, the temperature of the body decreases to 120°. How long will it take for the temperature to decrease to 90°? *[T-P, p.129]*

Example 5. *(exponential growth-death of a population.)* The death rate of an ant colony is proportional to the number present. If no births were to take place, the population at the end of one week would be reduced by one-half. Because of births (also at a rate proportional to the number present), the population doubles in two weeks. Determine the birth rate/ week. *[TP,132]*

Example 6. (*radioactive decay and composition of an ore.)* U-238 decays to Pb-210 (half-life: 4.51 Gyr), which decays to Pb-206 (ordinary lead, non-radioactive) with a half-life of 22yr. After a long time, one may assume the two disintegration rates are roughly equal (`radioactive equilibrium'), and that the rate of the first process is constant. If this rate is measured to be 100 dpm per gram of a sample of a lead ore, estimate the mass of U-238 per gram of the ore. *[Braun p. 18]*.

Example 7. *(radioactive decay and art CSI.)* Lead used in paint includes non-radioactive Pb-206, plus small amounts of radioactive lead (Pb-210, half-life 22 yr) and Ra-226, which decays to Pb-210 with half-life 1600 yr. After a long time since production of the paint (say, much longer than 22 yr.), the Pb-210 is in radioactive equilibrium, and the two decomposition rates are roughly equal; but if the paint is new, the rate of decay of Pb-210 should be much greater than that of Ra-226 (since the amount of Pb is much greater.) This can be used to determine whether a painting is at least 300 yr old, or a modern forgery. Suppose the decomposition rates of Pb-210 and Ra-226 are measured, and found to be 8.5 dpm/g and 0.8 dpm/g. If this painting were 300 yr old, what would be the implied decomposition rate of Pb-210 at the time the paint was produced? Is this number unreasonably large? (Use the result from example 6, assuming radioactive equilibrium between U-238 and Pb-210 at the time of production). *[Braun sect. 1.3, simplified]*