Problems on eigenfunctions and applications.

1. Find the eigenvalues and eigenfunctions for the following problems:
   (i) \[ y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(L) = 0; \]
   (ii) \[ y'' + \lambda y = 0, \quad y(0) - y'(0) = 0, \quad y(\pi) = 0. \]

2. Find the solution of the initial/boundary value problems:
   (i) \[ u_t = 2u_{xx}, \quad u(0, t) = u(2, t) = 0, \quad u(x, 0) = \sin(\pi x/2) + 3\sin(5\pi x/2); \]
   (ii) \[ u_t = 3u_{xx}, \quad u_x(0, t) = u_x(\pi, t) = 0, \quad u(x, 0) = 2\cos x + 5\cos 3x; \]
   (iii) \[ u_t = u_{xx} + u, \quad u(0, t) = u(\pi, t) = 0, \quad u(x, 0) = 3\sin 2x - 7\sin 4x. \]
   (iv) \[ u_{tt} - 4u_{xx} = 0, \quad u(0, t) = u(3, t) = 0, \quad u(x, 0) = \sin(\pi x/3), \quad u_t(x, 0) = 5\sin(\pi x) \]
   (v) \[ u_{tt} - 9u_{xx} = 0, \quad u_x(0, t) = u_x(\pi, t) = 0, \quad u(x, 0) = \cos 2x, \quad u_t(x, 0) = 4\cos 3x - 5\cos 5x. \]

3. Find the solution of the following problems as an infinite series; express the coefficients in the series in terms of integrals of \( f \) and \( g \).
   (i) \[ u_t - 7u_{xx} = 0, \quad u(0, t) = u(1, t) = 0, \quad u(x, 0) = f(x). \]
   (ii) \[ u_{tt} - 4u_{xx} = 0, \quad u_x(0, t) = u_x(\pi, t) = 0, \quad u(x, 0) = f(x), \quad u_t(x, 0) = g(x). \]