DIFFERENTIAL EQUATIONS-1ST. EXAM- SEPTEMBER 27,1994

1.[25]Solve the initial value problem:

$$xy' + y = e^x, \quad y(1) = 1.$$

Include the interval where the solution is defined. Remember to reduce to 'standard form' first!

2. [20]A population model is described by the differential equation:

$$\frac{dp}{dt} = -(1 - \frac{p}{2})(1 - \frac{p}{3})p.$$

(p in units of 10^3 individuals, t in years.)
(i) [7] Sketch the 'flow diagram' for the model;
(ii) [7] What is the asymptotic behavior of a population whose initial number of individuals is 2500?
(iii) [6] Are all solutions of the model guaranteed to exist for all $t > 0$? Explain.

3. [25]Consider the equation:

$$\frac{dy}{dx} = -\frac{2x + y}{x + 3y}.$$

(i) [10] Show the equation is exact;
(ii) [10] Find the solution of the initial value problem $y(1) = 1$ in implicit form.

4. [20]A swimming pool whose volume is 10,000 gal. contains water that is 0.01% chlorine. At $t = 0$, city water with 0.001% chlorine is pumped into the pool at 5 gal/min; the water in the pool flows out at the same rate. What is the concentration of chlorine in the pool after one hour? When will the concentration of chlorine in the pool equal 0.002% ?

5.[10]A building with no heating or cooling has a time constant of 2 h. Assuming the outside temperature varies as a sine wave with an average value of 60 F reached at 8 a.m. and a maximum of 80 F at 2 p.m., write down the differential equation describing the temperature inside the building as a function of time. (You do not have to solve the equation). Take the time origin ($t = 0$) to correspond to 8 a.m. , and the period of the sine wave to be 24h.

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