MATH 231- FALL 2003- HOMEWORK SET 3

1. Find the general solution and sketch a typical solution curve.
(a) $y'' - 2y' + 2y = 0$;
(b) $y'' + 6y' + 13y = 0$;
(c) $y'' - 6y' + 9y = 0$.

2. Use the method of reduction of order to find a second solution.
(a) $x^2y'' + 3xy' - 3y = 0, \quad x > 0; \qquad y_1(x) = x$.
(b) $(x-2)y'' - xy' + 2y = 0, \quad x > 2; \qquad y_1(x) = e^x$.

3. Find the solution to the non-homogeneous initial-value problems:
(a) $y'' + 9y = x^2 + 3e^x; \quad y(0) = 0; y'(0) = 2$;
(b) $y'' + 9y = \sin 3x; \quad y(0) = y'(0) = 0$.

4. Find the general solution (use variation of parameters):
(a) $y'' + y = 2\tan x, 0 < x < \pi/2$;
(b) $x^2y'' - 2y = 3x^2 - 1, \quad x > 0$ ($y(x) = x^2$ and $y(x) = 1/x$ solve the homogeneous equation.)

5. (From *Boyce-Di Prima*). A spherical block of radius l and mass density ρ is floating in a fluid of density ρ_0 ($\rho_0 > \rho$).If the block is depressed slightly and then released, it oscillates in the vertical direction. Derive the differential equation of motion and find the period of the oscillations (neglect damping).

6. (From *Nagle-Saff-Snyder*) Show that the *boundary value problem*:

$$y'' + \lambda^2 y = \sin t; \quad y(0) = 0; y(\pi) = 1$$

has a solution if and only if $\lambda \neq \pm 1, \pm 2, \pm 3, \ldots$

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