DIFFERENTIAL EQUATIONS-1ST. EXAM- OCTOBER 2, 2003

Instructions. Solve the following problems. No credit for answers without justification. Closed book, closed notes. Calculators not allowed. Good luck! **Time given:** 75min (11:10-12:25.)

1. [12pts. ea.] Find the general solution ($y = y(t)$ in each case):
(a) $y' + t^2 y = t^2$;
(b) $2tyy' + y^2 + 2t = 0$ (exact);
(c) $y' - 2y - 5y^3 = 0$ (Bernouilli);
(d) $y'' + 4y = \sin t$;
(e) $t^2 y'' - 3ty' + 3y = t, \quad t > 0$ (homogeneous eqn. is of Euler type).

2. [7 pts. ea.] Find the largest interval where the solution of the initial-value problem is defined; justify your answer. ($y = y(x)$ in each case.)
(a) $(\cos x)y' + (\ln x)y = 2x, \quad y(1) = 1$;
(b) $(x-2)y'' + xy' + y = 0, \quad y(1) = y'(1) = 0$;
(c) $y' + (y-1)(y-2) = 0, \quad y(0) = 3/2$.

3. For the autonomous first-order equation:

$$y' = y - y^{1/3}, \quad y = y(t):$$

(*items can be answered independently*)
(a) [3pts.] Find the constant solutions;
(b) [10 pts.] Find the general solution, assuming $y(0) > 1$. (*Hint:* let $v = y^{1/3}$);
(c) [6pts.] The functions $y_1(t) \equiv 0$ (for all t) and:

$$y_2(t) = 0 \text{ for } t \leq 0, y_2(t) = (1 - e^{2t/3})^{3/2} \text{ for } t \geq 0$$

both solve the initial-value problem defined by the equation and the initial condition $y(0) = 0$. Why doesn't this contradict the existence-uniqueness theorem for first-order equations?