

# EXAM 1

You must upload the solutions to this exam by 11:59pm on Sunday 07/21. Since this is a take home, I want all your solutions to be neat and well written.

**You can look at class discussions on Cocalc and *our* book only!** You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

You can use a computer only to *check* your answers, but **you need to show work in all questions.**

1) [15 points] Use the *Extended Euclidean Algorithm* to write the GCD of 235 and 185 as a linear combination of themselves. *Show the computations explicitly!* [**Hint:** You should get 5 for the GCD!]

2) [15 points] If  $a$  and  $b$  are positive integers such that  $ab = 3321$  and  $\gcd(a, b) = 3$ , then what is  $\text{lcm}(a, b)$ ?

3) [15 points] Let  $a$  and  $b$  be positive integers with  $(a, b) = d$ . Prove that  $(a/d, b/d) = 1$ .

4) [20 points] Find the remainder of  $10001 \cdot 674378^{584} - 3728382$  when divided by 5. *Show your computations explicitly!*

5) [20 points] Give the set of all integer solutions of the system

$$\begin{aligned}x &\equiv 4 \pmod{15}, \\3x &\equiv 11 \pmod{14}.\end{aligned}$$

6) [15 points] Prove that 1234567 is not a perfect square.