Exam 4

You must upload the solutions to this exam (as a PDF file) on Canvas by 11:59pm on Friday 07/06. Since this is a take home, I want all your solutions to be neat and well written.

You can look at *our* book only! You *cannot* look at our videos, solutions posted by me or *any* other references (including the Internet) without my previous approval. Also, of course, you cannot discuss this with *anyone*!

1) Let $f : A \to C$ and $g : B \to C$, with $A \cap B = \varnothing$. Prove that $(f \cup g) : A \cup B \to C$ [i.e., that $f \cup g$ is a function from $A \cup B$ to C].

[**Note:** This was a HW problem, but you cannot look at the solution for it, not even your own. You need to redo it.]

2) Let $f : A \to B$ and $g : B \to C$. Prove that if f is onto and g is not one-to-one, then $g \circ f$ is not one-to-one.

[**Note:** This was a HW problem, but you cannot look at the solution for it, not even your own. You need to redo it.]

3) We say that a function $f : A \to A$ [note it's from A to A itself!] has a *fixed point* if there is $a \in A$ such that $f(a) = a$. Let $g : A \to B$ an one-to-one and onto function. Prove that if $f : A \to A$ has a fixed point, then so does $g \circ f \circ g^{-1}$. [Note that $g \circ f \circ g^{-1} : B \to B$, so the fixed point is some $b \in B$.]

4) Prove *by induction* that for all $n \in \{0, 1, 2, 3, 4, \ldots\}$ we have that $3 \mid (25^n - 1)$ [i.e., 3 divides $25^n - 1$]. [There are other ways to prove this statement, but you must prove it by induction!]

5) Let

$$a_1 = 1,$$

$$a_n = \frac{a_{n-1}}{a_{n-1} + 1} \text{ for } n \geq 2.$$

Prove that $a_n = \dfrac{1}{n}$ for all $n \in \{1, 2, 3, \ldots\}$.

6) Let

$$a_1 = 4,$$

$$a_2 = 5,$$

$$a_n = (n-2) \cdot a_{n-2} + (n-3) \cdot a_{n-1} \text{ for } n \geq 3.$$

Prove that for all $n \geq 5$ we have that $a_n > 2^n$.