Midterm

M460 – Geometry

June 25th, 2012

1. Problem 2.12(c): Give a model of incidence geometry in which we have three [distinct] lines l, m and n such that $l \parallel m$ and $m \parallel n$, but $l \nparallel n$. [Give a concrete example of such lines in the given model!]

OLD 2. We will deal with the model of incidence geometry given by $\mathbb{Q}^2 = \{(x, y) \in \mathbb{R}^2 : x, y \in \mathbb{Q}\}$. Points are the elements of $\mathbb{Q}^2$ [i.e., points of $\mathbb{R}^2$ with both coordinates in $\mathbb{Q}$] and lines are given by the solutions [in $\mathbb{Q}^2$] of equations of the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Q}$. [You can assume that this is a model of incidence geometry without proving it.]

(a) Let $X = (0, 0)$, $Y = (1, 1)$, $A = (0, 1)$ and $B = (1, 0)$. [Draw a picture!] Show that A and B are on the *same side* of $\overleftrightarrow{XY}$. [**This statement is INCORRECT!**.]

(b) Show that Betweeness Axiom 4 does not hold for this model. [**Hint:** Consider $C = (0, -1)$. Show that B and C are on the same side of $\overleftrightarrow{XY}$, but A and C are on opposite sides of $\overleftrightarrow{XY}$, contradicting part (i) of the axiom.] **Even if you did not do part (a), you can use it to try to do part (b)!**

NEW 2. We will deal with the model of incidence geometry given by $\mathbb{Q}^2 = \{(x, y) \in \mathbb{R}^2 : x, y \in \mathbb{Q}\}$. Points are the elements of $\mathbb{Q}^2$ [i.e., points of $\mathbb{R}^2$ with both coordinates in $\mathbb{Q}$] and lines are given by the solutions [in $\mathbb{Q}^2$] of equations of the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Q}$. [You can assume that this is a model of incidence geometry without proving it.] Prove that $\mathbb{Q}^2$ does not satisfy the Congruence Axiom 1.

Continue on next page!

3. **Problem 2.15(a):** Let M be a model of incidence geometry in which every line has at least three distinct points in it. We will show that there are four point in this model with no three of them on the same line.

   (i) Start with three points, say A, B, C, not on the same line [by I-3]. Thus, one can easily prove that lines $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$ are distinct. [You can use it without proving it!]

   (ii) By hypothesis, line $\overleftrightarrow{AB}$ has a third point, say B', and line $\overleftrightarrow{AC}$ has a third point, say C'.

   (iii) Consider the line $\overleftrightarrow{B'C'}$. By hypothesis there is a third point on it, say D.

   [Draw a picture!]

   (a) Prove that $\overleftrightarrow{B'C'} \neq \overleftrightarrow{AB}$. [In a similar way, you could prove that $\overleftrightarrow{B'C'} \neq \overleftrightarrow{BC}, \overleftrightarrow{AC}$. **You may use this fact in the next parts!**]

   (b) Prove that A, B and D are not on the same line. [In a similar manner you could prove that no three among A, B, C and D are in the same line, finishing the proof of 2.15(a).] **Even if you did not do part (a), you can use it to try to do part (b)!**

4. **Problem 3.9:** Given a line l, a point A on l and a point B not on l, prove that every point of $\overrightarrow{AB}$ except A is on the same side of l as B. [**Hint:** Use an RAA argument. You may also use the fact that if X, Y and P are colinear, then $P \notin \overrightarrow{XY}$ is equivalent to $P * X * Y$.]