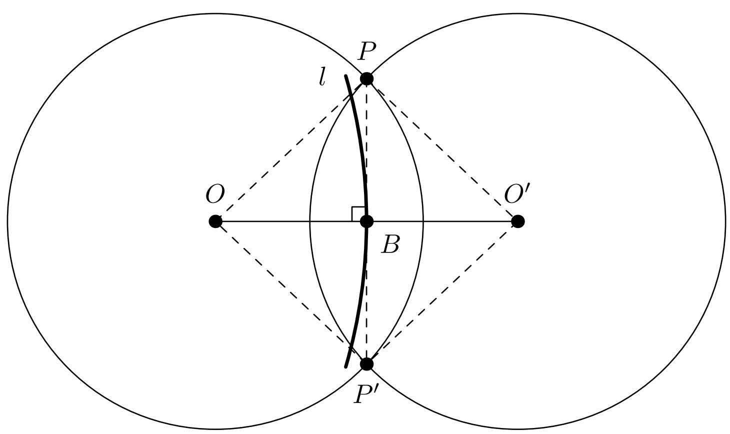
Final

M460 – Geometry

July 3rd, 2012

1. [This is the homework from yesterday.] Assume we have two circles, with centers O and O' and same radius, say OR, which intersect in two distinct points, say P' and P. Let B be the midpoint of OO' and l be the line through B perpendicular to $\overleftrightarrow{OO'}$. Assuming that P and P' are in opposite sides of $\overleftrightarrow{OO'}$, show that $P, P' \in l$. **You cannot use any continuity principle!** [I.e., no Circle-Circle, Line-Circle, Segment-Circle, Dedekind's Axiom, etc.] Note that we do *not* know, at least at first, if B, P and P' are colinear [as the picture seems to indicate], so don't use it!

[**Hint:** Melinda was on the right track. Use congruence of triangles to show that $\overleftrightarrow{PP'} \perp \overleftrightarrow{OO'}$ and $B \in \overleftrightarrow{PP'}$. This should help!]



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2. Show by giving explicit counterexamples [well drawn pictures, preferably explicitly specifying the radii and centers of circles] that the following statements of Euclidean Geometry *do not hold* in the upper half plane (UHP).

(a) *"There can be no line entirely contained in the interior of angle."*

(b) Remember that circles in the UHP are Euclidean circles entirely contained in the upper half plane [but the real center is below the Euclidean center]. *"Given three non-colinear points, there is a circle passing through all of them."*

[**Hint:** There are a couple of different ways to do this. Given three non-colinear points on the UHP, if there is a [non-Euclidean] circle through them, then it is also an Euclidean circle through them *entirely* contained in the UHP. So, if there is no circle through the three non-colinear points, then either there is an Euclidean circle through the points, but it is not contained in the UHP, or there is no [Euclidean] circle at all through the three points.]

3. Consider the distorted model of Problem 35 on pg. 152 [presented in the second project yesterday], where distances on the x-axis are twice as long as they are in the usual $\mathbb{R}^2$ model. [Everything else is the same.]

(a) Give an example of a triple (x, y, z) which represents the lengths of three sides of a triangle that exists only if one of its sides is on the x-axis. [**Hint:** *Triangle Inequality* on pg. 171.]

(b) Give examples [with pictures] of rays $\overrightarrow{AB}$ and $\overrightarrow{CD}$ and a circle γ, such that A and C in the interior of γ, $\overrightarrow{AB}$ does not intersect γ and $\overrightarrow{CD}$ intersects γ in exactly two points.

4. Prove that Hilbert's Euclidean Parallel Postulate is equivalent to the transitivity of parallels, i.e., "if $l \parallel m$ and $m \parallel n$, then $l \parallel n$".

[**Hint:** Use Proposition 4.7. In other words, it suffices to show that transitivity of parallels is equivalent to "if $l \parallel m$ and t intersects l, then t also intersects m".]