May 7th, 2018

Math 351

Name:...

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Spring 2018

Student ID (last 6 digits): XXX- ....

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 13 printed pages (including this one and two pages for scratch work in the end).

No books or notes are allowed on this exam!

Questions 3, 4 and 5 will make up your Midterm 2 grade.

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	20	
8	20	
Total	100	

1) [10 points] Find the remainder of $493438 + 76584576 \cdot 47300272^{1000}$ when divided by 5.

2) [10 points] Let $n \in \mathbb{Z}$. Prove that $(n, n+1) = 1$.
[**Note:** This was a HW problem.]

3) [10 points] Find all $x \in \mathbb{Z}$ satisfying [simultaneously]:

$$\begin{aligned} x &\equiv 1 \pmod{7}, \\ x &\equiv 4 \pmod{11}. \end{aligned}$$

If there is no such x, simply justify why.

4) [10 points] Prove that the only subring of $\mathbb{F}_p$ [i.e., of $\mathbb{Z}/p\mathbb{Z}$] is itself.
[**Note:** It was a HW problem that the only subring of $\mathbb{Z}$ was itself. This is similar.]

5) Below are the factorization of $f, g \in \mathbb{F}_3[x]$ into distinct irreducibles.

$$f = x \cdot (x+1)^3 \cdot (x^2+1) \cdot (x^2+x+2)^4$$
$$g = 2 \cdot x^2 \cdot (x+2)^2 \cdot (x^2+1)^3 \cdot (x^2+x+2)$$

(a) [4 points] Does $g \mid f$? [*Justify!*]

(b) [3 points] Give the factorization of the $\gcd(f, g)$.

(c) [3 points] Give the factorization of $\operatorname{lcm}(f, g)$.

6) Examples:

(a) [5 points] Give an example of an *infinite* commutative ring which is *not* a domain.

(b) [5 points] Give an example of a field properly containing $\mathbb{R}$ [i.e., contains $\mathbb{R}$ but it is not $\mathbb{R}$ itself], but not containing $\mathbb{C}$. [Note that this excludes $\mathbb{C}$ itself.]

7) Determine if the polynomials below are irreducible or not in the corresponding polynomial ring. *Justify each answer!*

(a) [3 points] $f = x^{2018} - x + 2018$ in $\mathbb{R}[x]$.

(b) [3 points] $f = x + \pi$ in $\mathbb{C}[x]$.

(c) [3 points] $f = x^7 + 110x^5 + x^2 + 97x$ in $\mathbb{F}_{521}[x]$.

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(d) [3 points] $f = 3x^7 + 6x^6 - 9x^4 + 120x^3 - 15x + 2$ in $\mathbb{Q}[x]$.

(e) [4 points] $f = 64x^3 - 3x^2 + 32x + 30001$ in $\mathbb{Q}[x]$.

(f) [4 points] $f = x^3 + 2x^2 - 2x - 1$ in $\mathbb{Q}[x]$.

8) Let $\sigma, \tau \in S_9$ be given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 1 & 5 & 4 & 3 & 9 & 2 & 8 & 6 \end{pmatrix} \quad \text{and} \quad \tau = (1\,3\,8)(2\,4\,5\,9).$$

(a) [3 points] Write the *complete* factorization of σ into disjoint cycles.

(b) [3 points] Compute σ^{-1}. [Your answer can be in any form.]

(c) [3 points] Compute $\tau\sigma$. [Your answer can be in any form.]

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(d) [3 points] Compute $\sigma\tau\sigma^{-1}$. [Your answer can be in any form.]

(e) [3 points] Write τ as a product of transpositions.

(f) [2 points] Compute $\text{sign}(\tau)$.

(g) [3 points] Compute $|\tau|$ (the order of τ in S_n).

Scratch:

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