April 18th, 2018

Math 351

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Spring 2018

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 4

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No calculators, books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) [25 points] Let $f = x^5 + x^4 + x^2 + 2x + 1$ and $g = x^4 + 2x$ in $\mathbb{F}_3[x]$. Find $\gcd(f, g)$ [in $\mathbb{F}_3[x]$].

2) [25 points] Let R be a domain. Prove that $f \in R[x]$ is a unit [of $R[x]$] if and only if f is a unit of R [i.e., f is a constant polynomial and this constant is a unit of the ring R]. [**Note:** This was a HW problem.]

3) [25 points] Prove that if R is a domain, then $R[x]$ is also a domain. [You can assume that $R[x]$ is already a commutative ring, as polynomial rings always are (under the assumption R is commutative).]

[**Note:** This was done in class.]

4) [25 points] Let F be a field and $f, g \in F[x]$ such that $f \cdot g = a \cdot x^n$, for some $a \in F$, $a \neq 0$, and $n \in \mathbb{Z}_{>0}$. Prove that $f = b \cdot x^r$, $g = c \cdot x^s$ such that $bc = a$ and $r + s = n$.
[**Hint:** Section 3.6.]

Scratch: